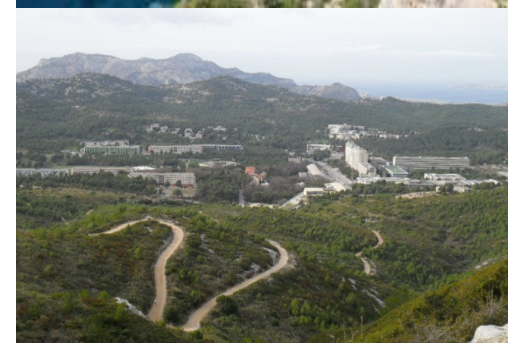
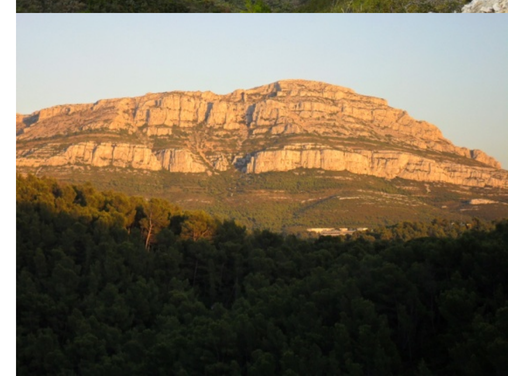
Samarkand
2019

T. Martin

Centre de Physique Théorique
Aix Marseille Université

Quantum transport in hybrid topological superconductor junctions 1 + 2



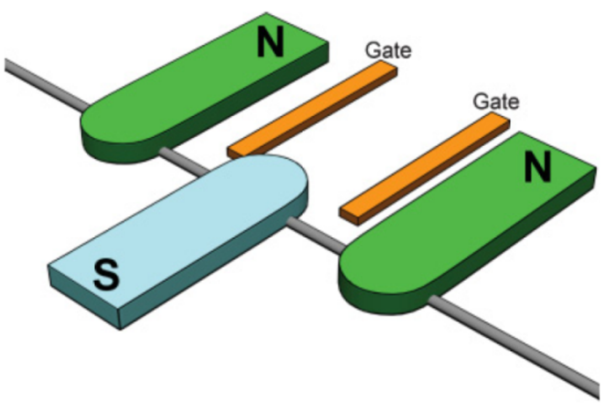



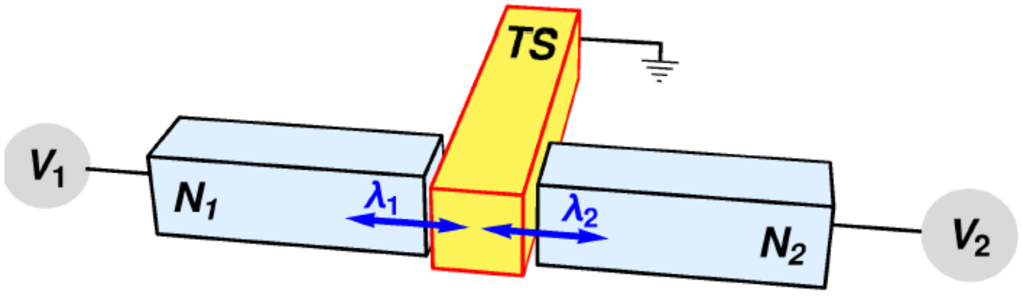
Collaborators (present and past):

- T. Jonckheere, J. Rech, L. Raymond (CPT, this course)
- A. Zazunov, R. Egger (HHU Dusseldorf, this course)
- A. Levy Yeyati (U. Autonoma de Madrid, this course)

- D. Feinberg (Néel Lab, past work on BCS noise)
- G. Lesovik (Landau Institute, past work on BCS noise)
- R. Landauer (IBM, past work on normal metal noise)

- D. Chevallier (PhD student)
- D. Bathellier (Masters student)





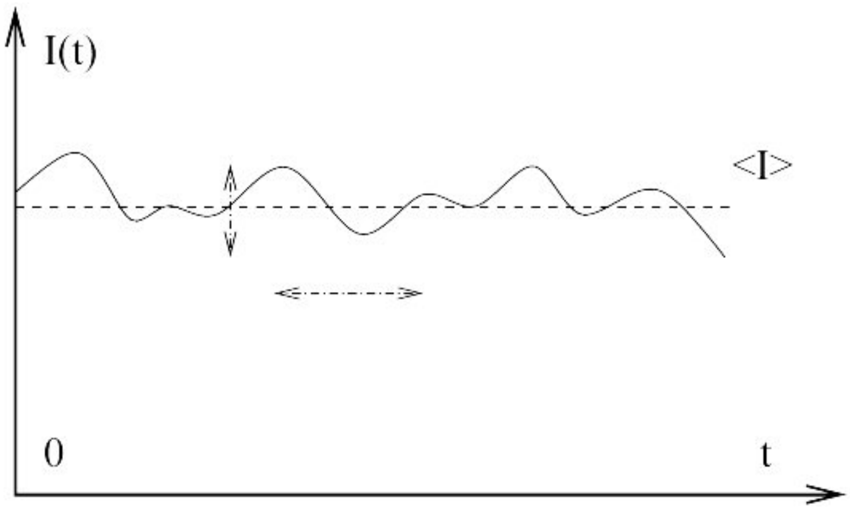
OUTLINE:

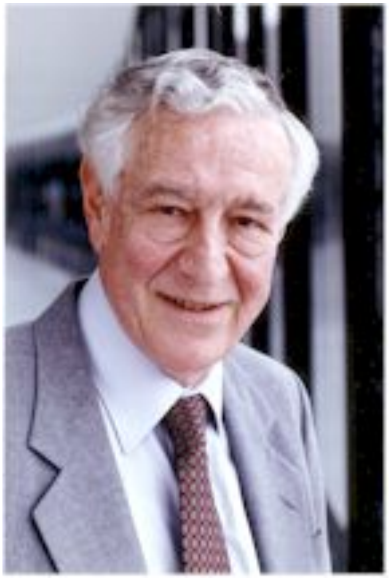
1) Introduction to noise in mesoscopic devices
2) Current and noise in BCS hybrid devices: the Cooper pair beam splitter
3) Introduction to topological superconductors and Majorana fermions
4) **Hanbury Brown and Twiss noise correlation with a topological superconductor beam splitter**
5) **Finite frequency noise of a normal metal / topological superconductor beam splitter (introduction to finite frequency noise)**
6) **« Giant » noise in a junction between three topological superconductors.**
7) Conclusions.

1) Introduction to noise in mesoscopic physics

Measurable quantities: current and noise

« the noise is the signal »



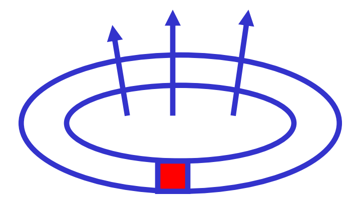


R. Landauer

$$S_{ij}(\omega)=\lim_{T\to+\infty}\frac{1}{T}\int_{-T/2}^{T/2}dt\int_{-\infty}^{+\infty}dt'\,e^{i\omega t'}\left(\langle I_i(t)I_j(t+t')\rangle-\langle I_i\rangle\langle I_j\rangle\right).$$

Quantum Mesoscopic Physics/ Nanophysics:

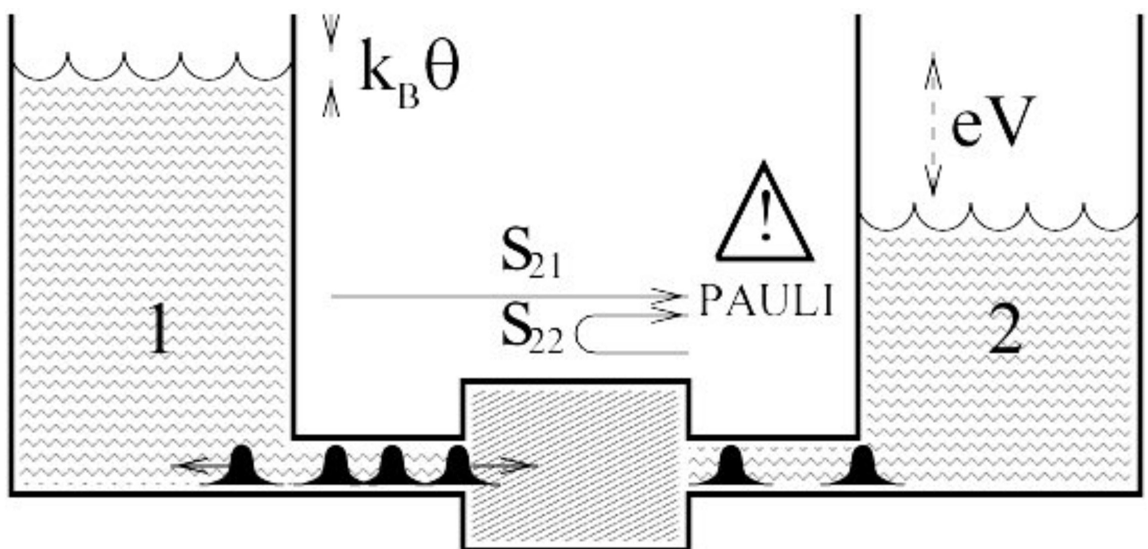
Closed systems in thermal equilibrium (Josephson effect, persistent current, level statistics in quantum dots…)



Open systems, with a bias imposed.

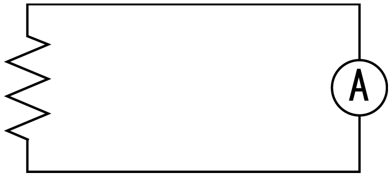
•Scattering theory (non interacting case mostly) with large e-reservoirs (Landauer Buttiker)

•Hamiltonian approach: use non-eq technique (Keldysh): Nozières…



Johnson-Nyquist noise and Shot noise

① Johnson-Nyquist noise for equilibrium circuit

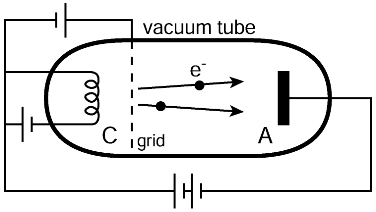


$$S_{II} \sim 4k_{\mathrm{B}}T/R$$

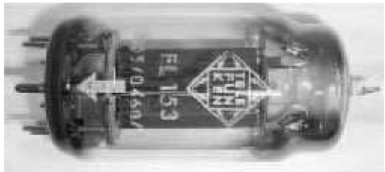
information about resistance & temperature
… just disturbance

Harry Nyquist
(1889-1976: U.S.)

② Shot noise in a vacuum tube



Electrons are emitted by thermal agitation



(the « best » thermometer for measuring electron temperature)

Shot noise: Schottky formula

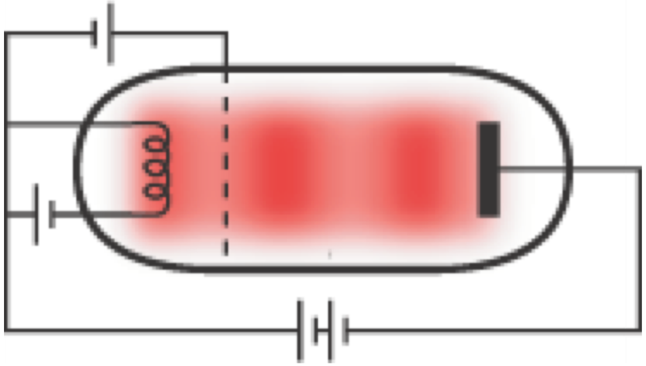
noise power $S_{II}(\omega) \sim 2e\bar{I}$

"simple way to measure the charge of electron"

Annals der Physik (1918)

Walter Schottky
(1886-1976: Germany)

classical picture of current

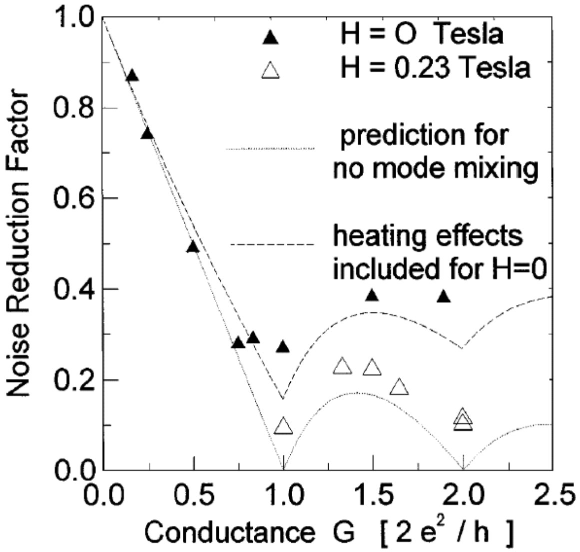


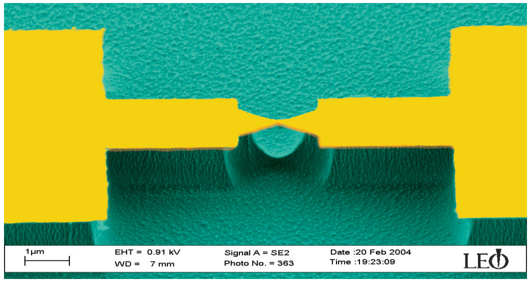
Electrons are emitted

Independently from each other:

Poissonian process.

Quantum noise reduction





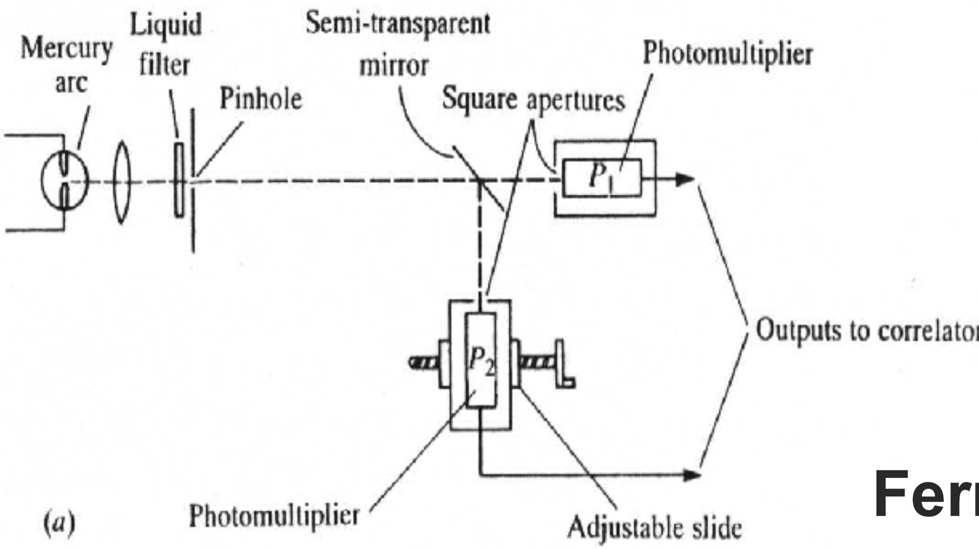
Fano factor

$$S(\omega=0)=\frac{e^3}{h}T(1-T)V \qquad F\equiv\frac{S(\omega=0)}{eI}=1-T$$

$$S_{LL}(0)=\frac{4e^2}{h}\left[2k_B\Theta\sum_\alpha T^2+eV\coth\left(\frac{eV}{2k_B\Theta}\right)\sum_\alpha T(1-T)\right]$$

Thermal-shot noise crossover

Hanbury Brown and Twiss experiment



Bunching effect: positive correlations

for thermal photons

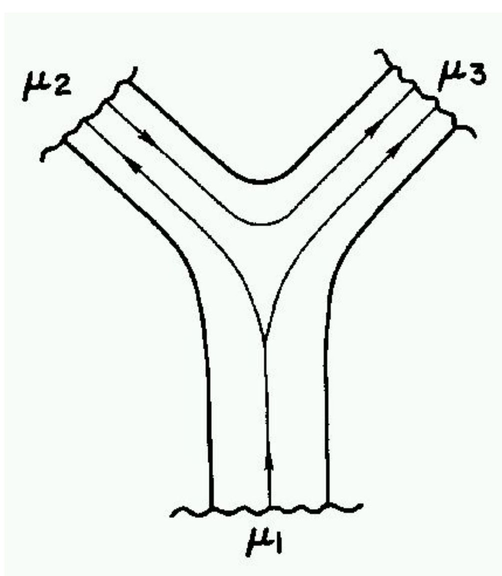
Fermions in nanophysics:

Negative correlations

T.M.+R. Landauer

M. Buttiker,

Phys Rev B 's 92)

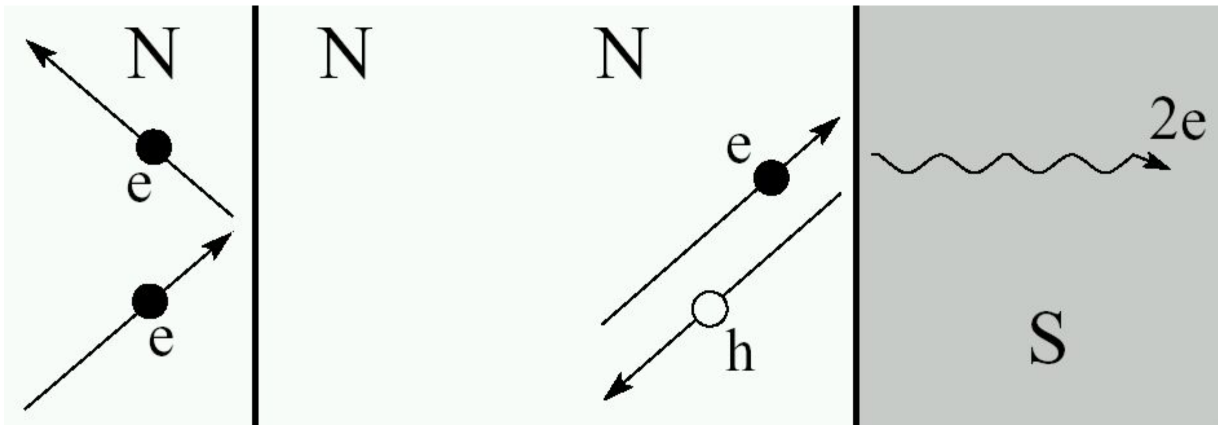


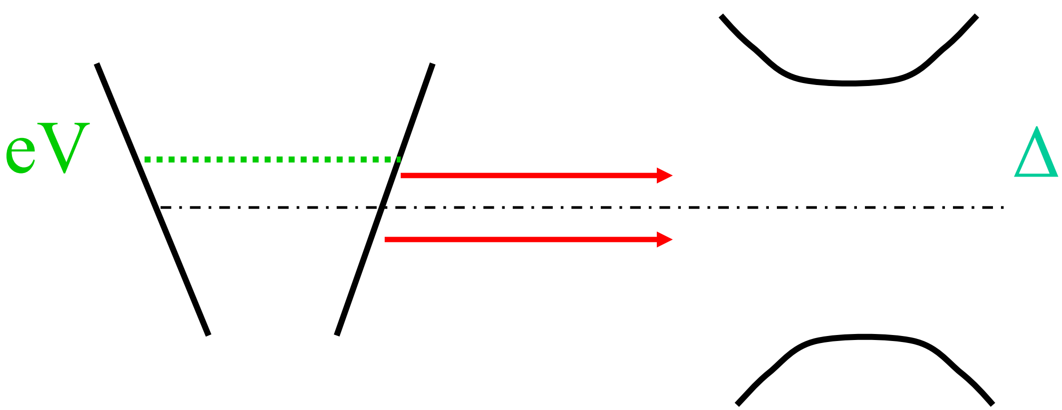
Experiments using quantum point contact: Schonenberger, Yamamoto (Science 99)

2) Noise correlations with a Cooper pair beam splitter

Mesoscopic superconductivity

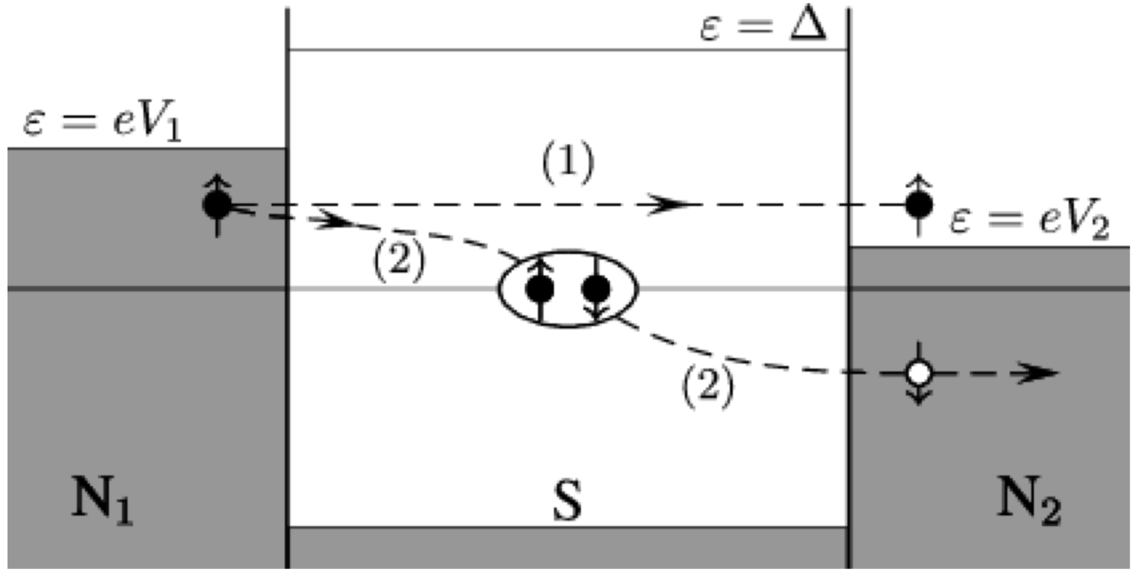
Andreev reflection: an electron is reflected as a hole at the boundary of a normal metal – superconductor junction





Equivalently: 2 electrons are transmitted from N as a Cooper pair in S   (Andreev JETP 60's, Blonder, Tinkham Klapwik PRB 80's)

Crossed Andreev Reflection



Crossed Andreev Reflection:

- An electron from N1 is transmitted through S as a hole in N2
- Equivalently, two electrons incident from N1 and N2 create a Cooper pair in S.

Superconducting source connected to normal leads:

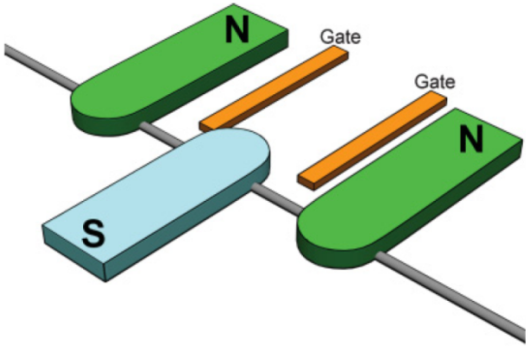
Noise crossed correlations > 0 or < 0

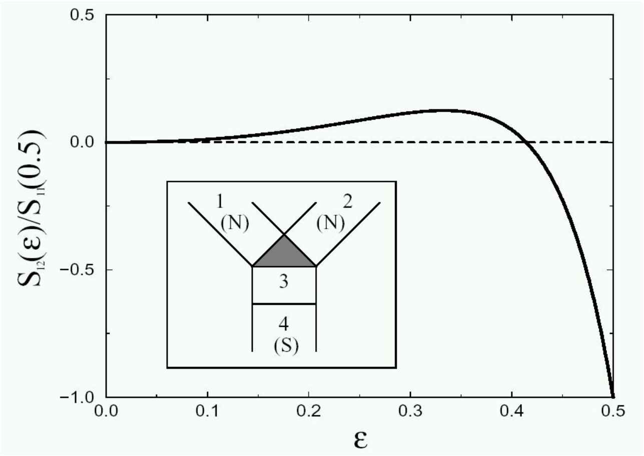
Martin, Phys Lett. A 1996, Anantram Data PRB96,

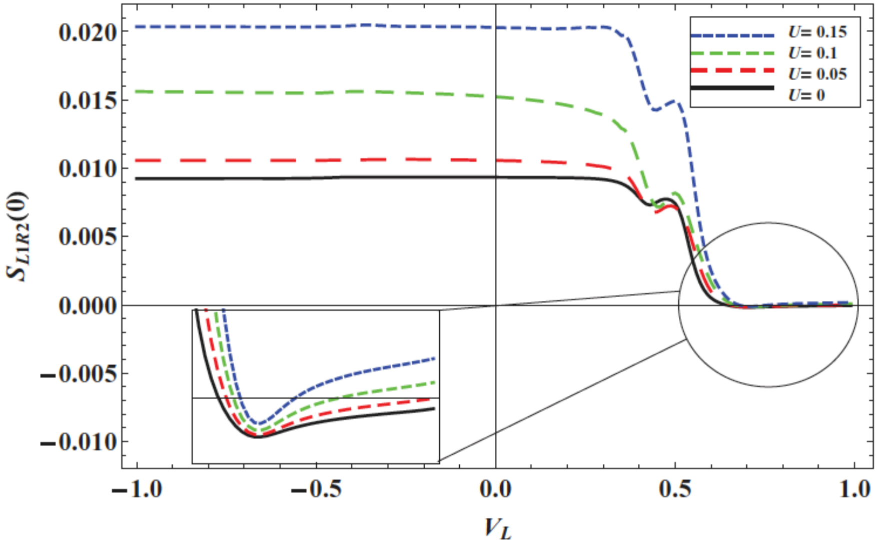
Torrès Martin EPJB 1999

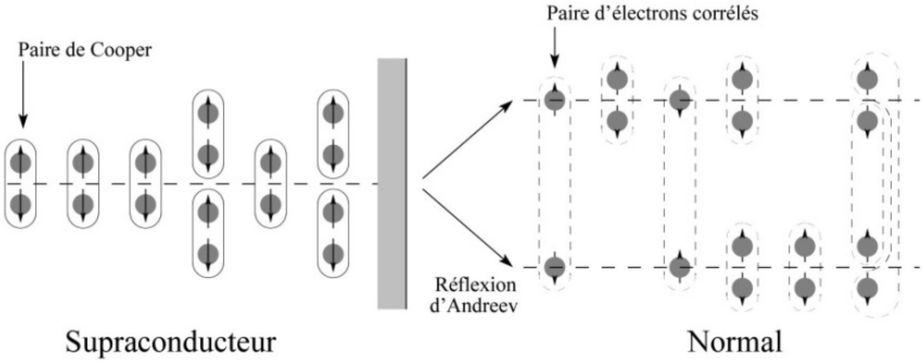
Chevallier PRB 2011

Rech PRB 2012





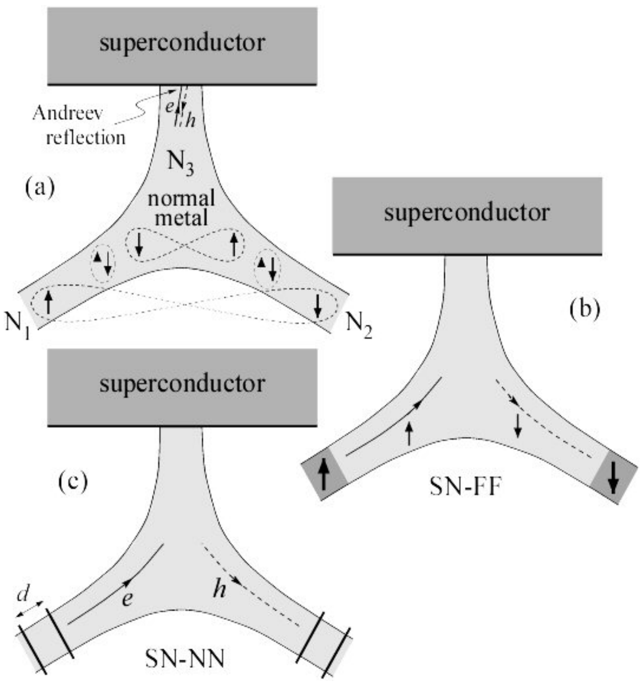


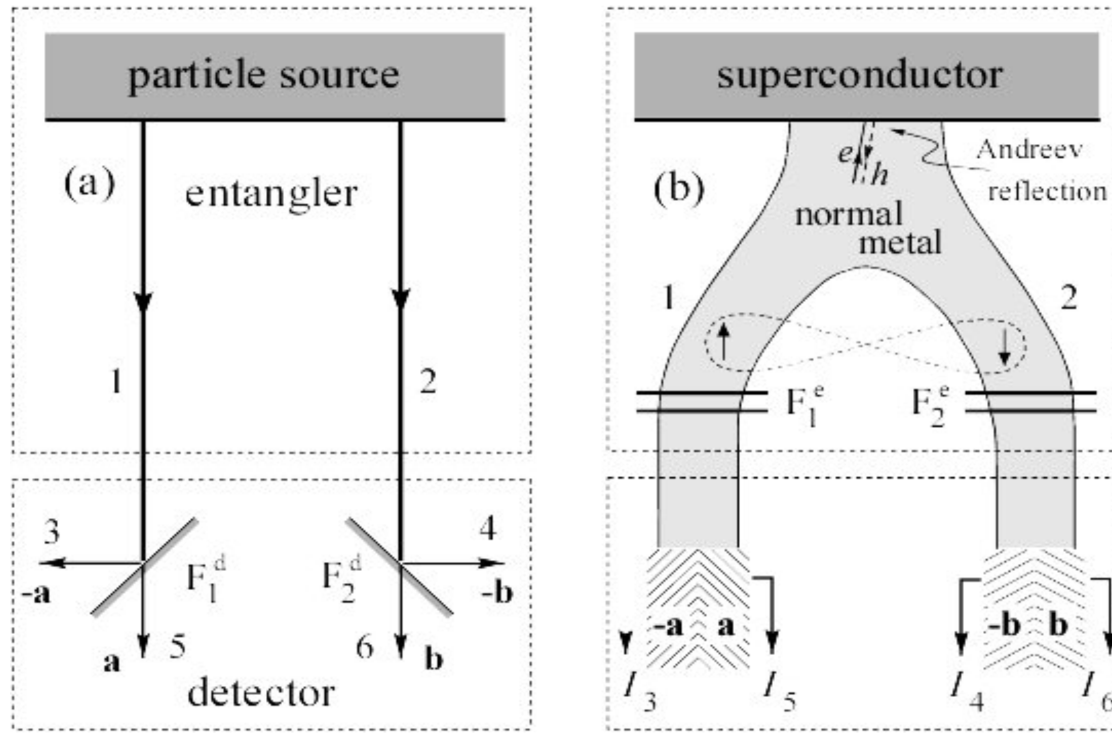


Superconductor: a source of entangled electrons

Lesovik Martin Blatter EPJB 2001 Recher, Sukorukov, Loss PRB 2001

Chtchelkatchev et al. PRB 2002 (Bell inequalities test from noise correlations)





Only positive noise cross correlations for energy filters or spin filters

Also Börlin, Belzig, Bruder PRL 02 **FCS**
Samuelsson Buttiker Chaotic PRL 02

Experimental evidence for CAR

Karlsruhe

VOLUME 93, NUMBER 19 PHYSICAL REVIEW LETTERS week ending 5 NOVEMBER 2004

Evidence for Crossed Andreev Reflection in Superconductor-Ferromagnet Hybrid Structures

D. Beckmann* and H. B. Weber
Forschungszentrum Karlsruhe, P.O. Box 3640, D-76021 Karlsruhe, Germany
H. v. Löhneysen
Forschungszentrum Karlsruhe, P.O. Box 3640, D-76021 Karlsruhe, Germany,
es Institut, Universität Karlsruhe, D-76128 Karlsruhe, Germany
ceived 16 April 2004; published 4 November 2004)

PRL 95, 027002 (2005) PHYSICAL REVIEW LETTERS week ending 8 JULY 2005

Experimental Observation of Bias-Dependent Nonlocal Andreev Reflection

S. Russo, M. Kroug, T. M. Klapwijk, and A. F. Morpurgo
Kavli Institute of Nanoscience, Delft University of Technology, Lorentzweg 1, 2628 CJ Delft, The Netherlands
(Received 21 January 2005; published 8 July 2005)

Delft

PRL 97, 237003 (2006) PHYSICAL REVIEW LETTERS week ending 8 DECEMBER 2006

Nonlocal Correlations in Normal-Metal Superconducting Systems

P. Cadden-Zimansky and V. Chandrasekhar
Department of Physics and Astronomy, Northwestern University, Evanston, Illinois 60208, USA
(Received 18 May 2006; published 6 December 2006)

…Evanston

nature Vol 461|15 October 2009|doi:10.1038/nature

LETTERS

Basel

Cooper pair splitter realized in a two-quantum-dot Y-junction

L. Hofstetter[1]*, S. Csonka[1,2]*, J. Nygård[3] & C. Schönenberger[1]

PRL 104, 026801 (2010) PHYSICAL REVIEW LETTERS week ending 15 JANUARY 2010

Carbon Nanotubes as Cooper-Pair Beam Splitters

Paris/Regensburg

L. G. Herrmann,[1,2,5] F. Portier,[3] P. Roche,[3] A. Levy Yeyati,[4] T. Kontos,[1,2,*] and C. Strunk[5]

Evidence of entangled electrons born from Cooper pairs splitting via current and noise correlations

Rehovot

Anindya Das, Yuval Ronen, Moty Heiblum,*
Diana Mahalu, Andrey V. Kretinin and Hadas Shtrikman

Evidence of entangled electrons born from Cooper pairs splitting via current and noise correlations

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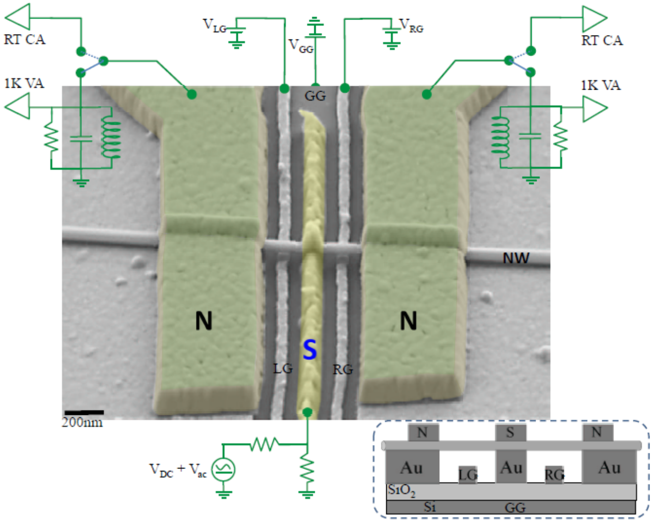
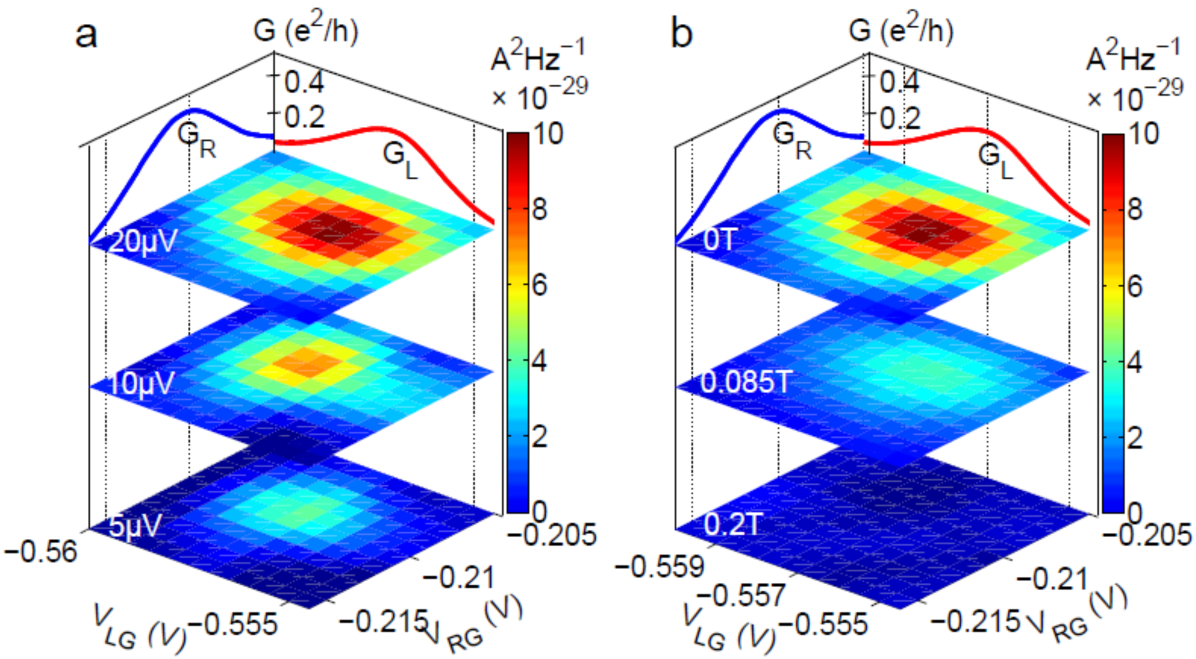


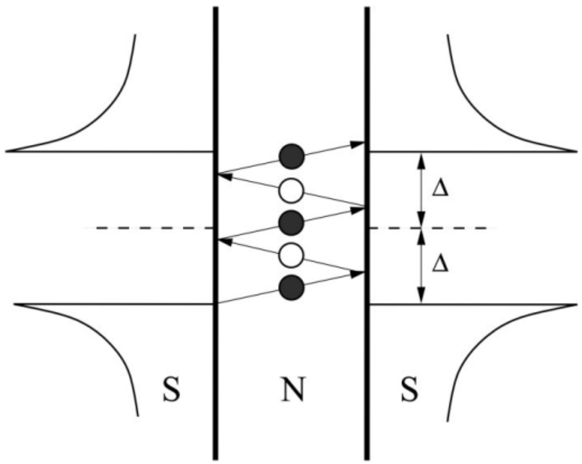


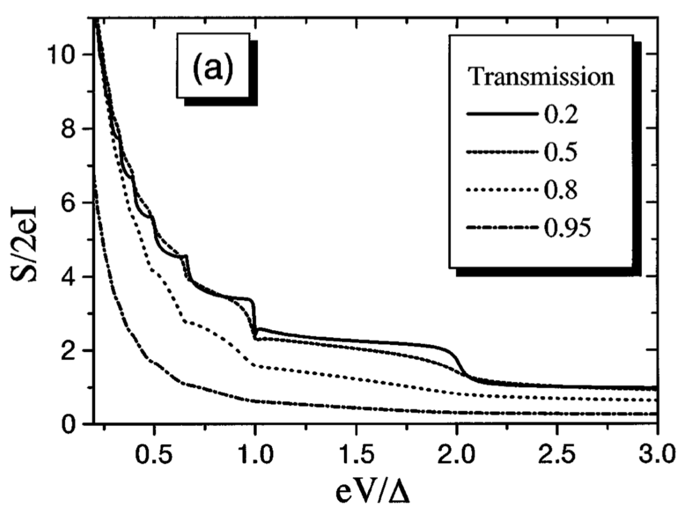
Positive noise cross correlations

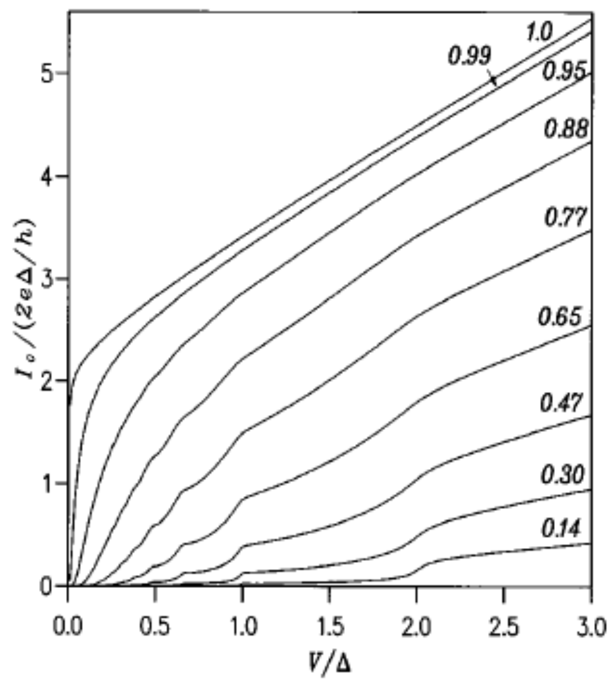
2 or more superconductors with a voltage bias:

Current and noise bear all harmonics of the Josephson frequency

DC contributions: Multiple Andreev Reflections (MAR)



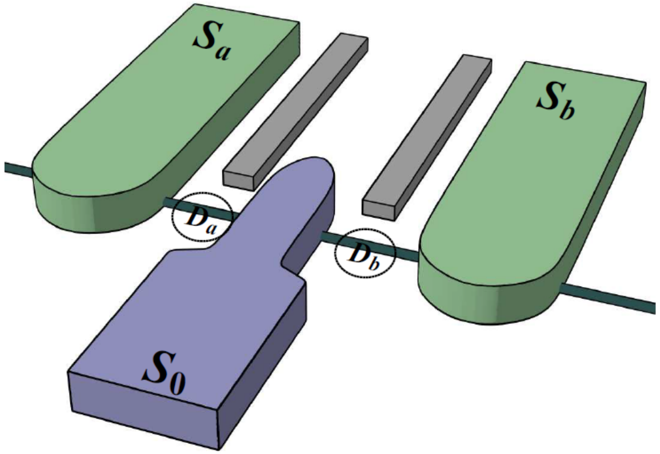




MAR steps at $eV = 2\Delta/n$

Cuevas PRB96, PRL99

Jonckheere et al. PRB 2013: superconductor bi-junction off equilibrium (all superconducting device)

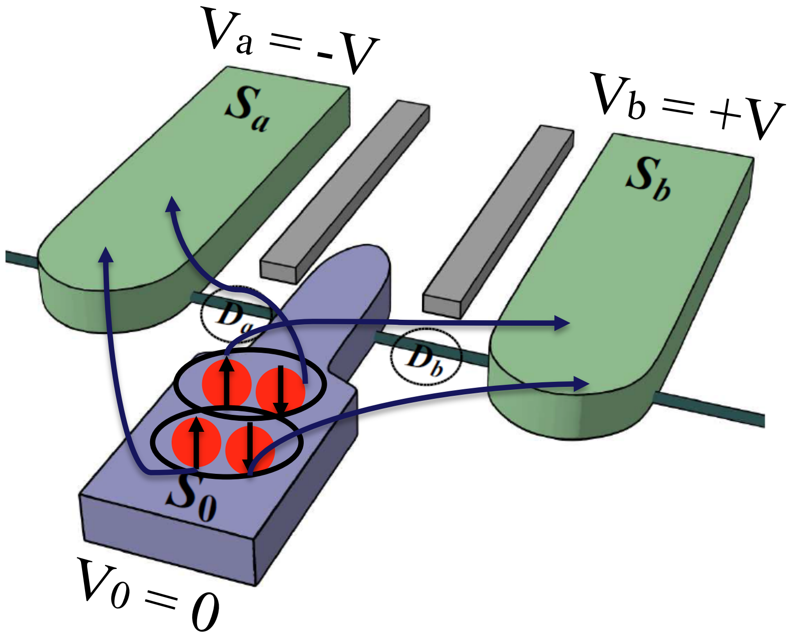


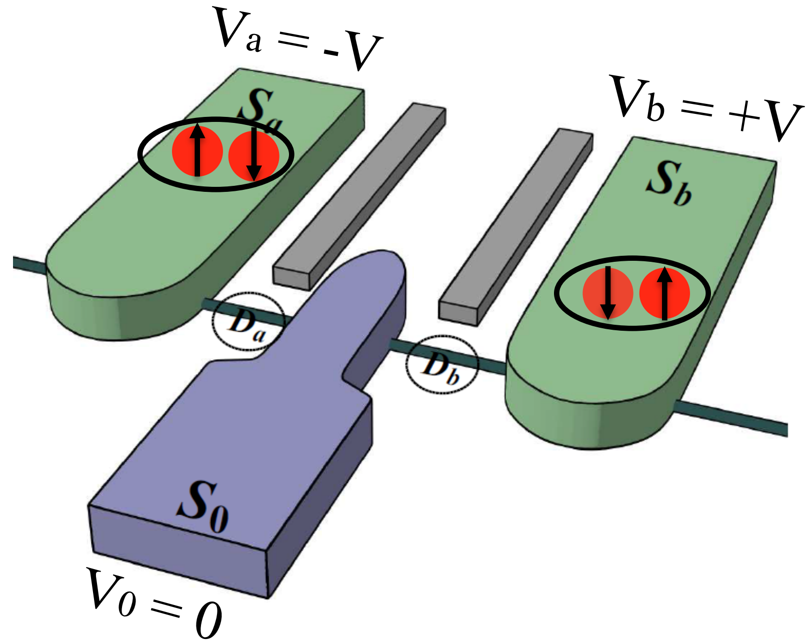
« Multiple Cooper Pair Resonnances »

3 Superconductors separated by quantum dots
Dots (generated by nanowires) between pairs of superconductors (S)
Phases applied on each S
Commensurate voltages → DC Josephson resonance

→ DC Josephson signal dependant on linear combinations of the 2 phase differences

The « Quartet » process



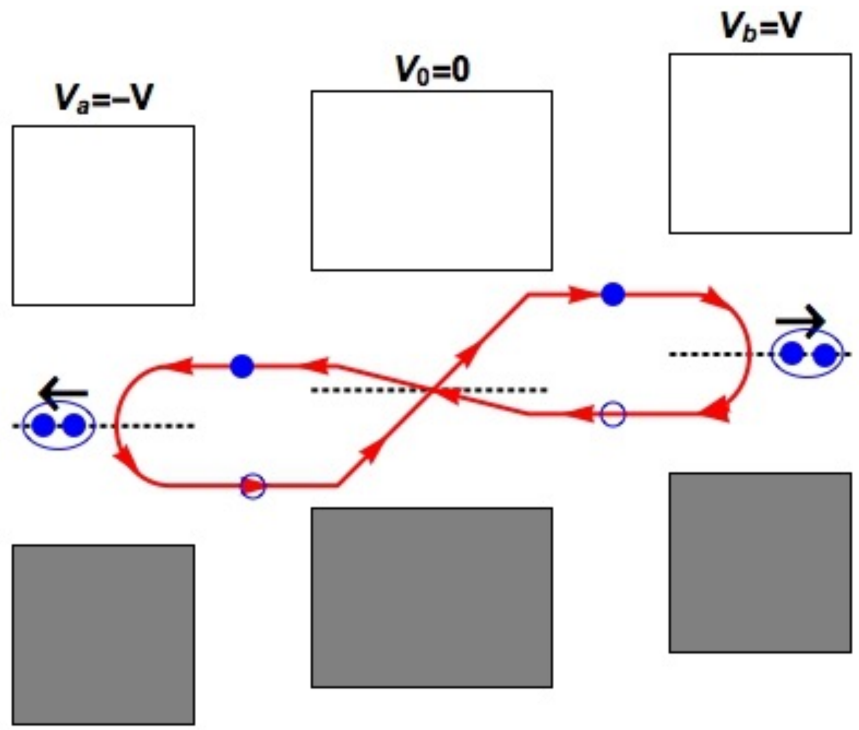


Initial State :
2 Cooper pairs at V=0

Final State :
1 pair at +V, 1 pair at -V

Energy conserving process
Transfer of 2 Cooper pairs
« quartet » of electrons

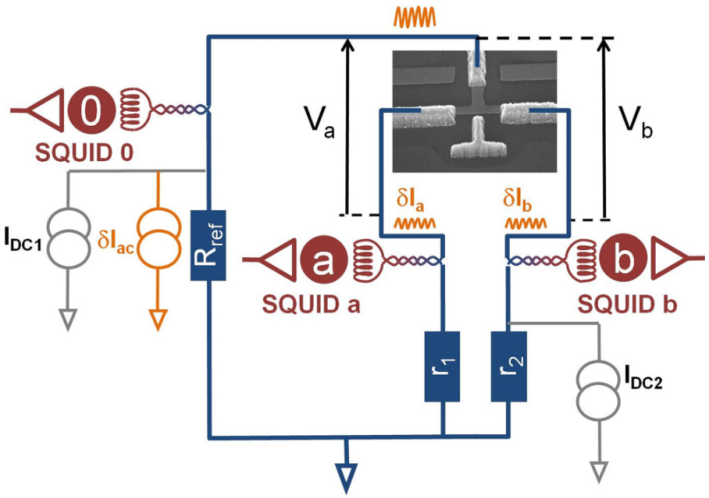
Energy Diagram of the Quartet process

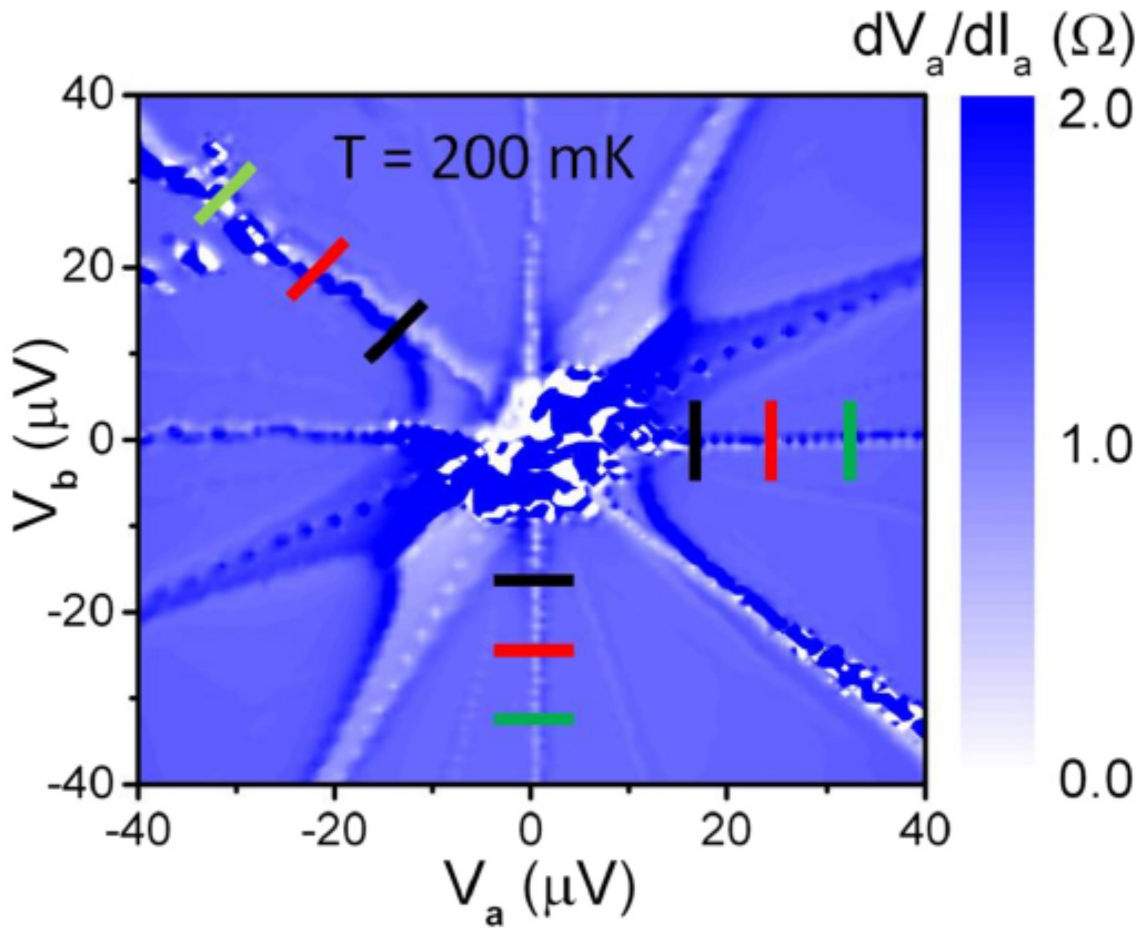


Combination of 2 crossed Andreev reflections in the central superconductors, and 2 standard Andreev reflections

Possible first evidence of multiple pair resonances

Subgap structure in the conductance of a three-terminal Josephson junction
A.H. Pfeffer, J. E. Duvauchelle,H. Courtois, R. M´elin, D. Feinberg, F. Lefloch
PRB **90**, 075401 (2014)





3) Introduction to topological superconductors and Majorana fermions

Ettore Majorana (1906-1938)

1906 Born in Catania, Italy
1928 The first paper on atomic spectroscopy
1933 The first idea on neutrons
(Fermi recommended publication, but not published. The credit was given to Chadwick.)
1933 Collaboration with Heisenberg at Germany
1933-37 Nothing published
1937 The last 9th paper on Majorana particles
1938 Jan. Chair in Theoretical Physics at Naples
1938 Mar. Mysterious disappearance

Enrico Fermi wrote:

> There are many categories of scientists: people of second and third rank, who do their best, but do not go very far; there are also people of first class rank, who make great discoveries, fundamental to the development of science. But then there are the geniuses, like Galileo and Newton. <u>Well Ettore Majorana was one of them.</u>

Majorana Fermions

Majorana's idea: Another form of the Dirac equation is possible

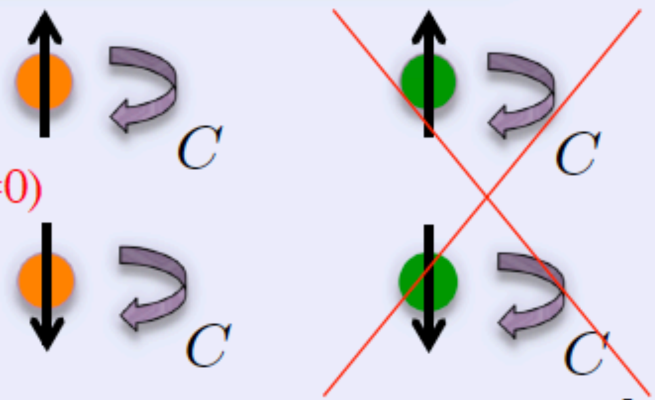
Real matrix

$$(i\tilde{\gamma}^{\mu}\partial_{\mu} - m)\tilde{\psi} = 0$$

$$\tilde{\psi} = \begin{pmatrix} \tilde{\psi}_0 \\ \tilde{\psi}_1 \\ \tilde{\psi}_2 \\ \tilde{\psi}_3 \end{pmatrix}$$

C: charge $q \rightarrow$ -q

Neutral particles (q=0) are only allowed!



Two kinds of Fermions appear! We can remove one of two.

C : Charge-conjugate operation (particle ↔ anti-particle)

$$\tilde{\gamma}^0 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & -i & 0 \\ 0 & i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix} \quad \tilde{\gamma}^1 = \begin{pmatrix} 0 & 0 & i & 0 \\ 0 & 0 & 0 & i \\ i & 0 & 0 & 0 \\ 0 & i & 0 & 0 \end{pmatrix}$$

$$\tilde{\gamma}^2 = \begin{pmatrix} i & 0 & 0 & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & -i & 0 \\ 0 & 0 & 0 & -i \end{pmatrix} \quad \tilde{\gamma}^3 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix}$$

A Majorana particle is its own anti-particle.

Search of Majorana particles in condensed-matter physics

Up to now, Majorana particles are not found in elementary-particle physics.

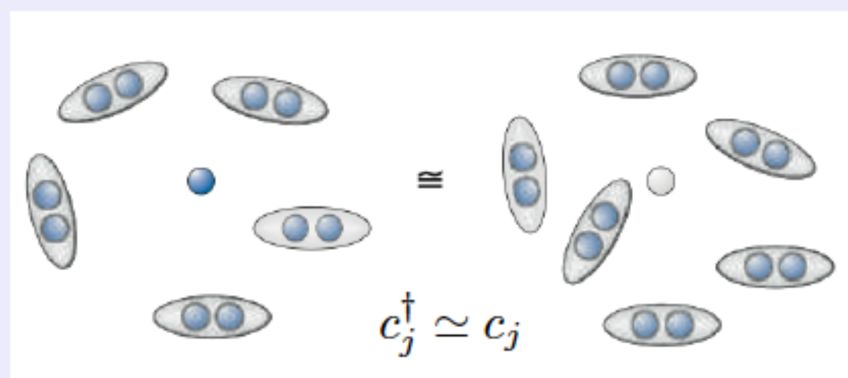
How about condensed-matter physics?

- Electron $c_j^\dagger$ $\rightarrow$ Particle
- Hole c_j $\rightarrow$ Anti-particle

C : Charge-conjugate op. $\rightarrow$ Electron-hole transformation

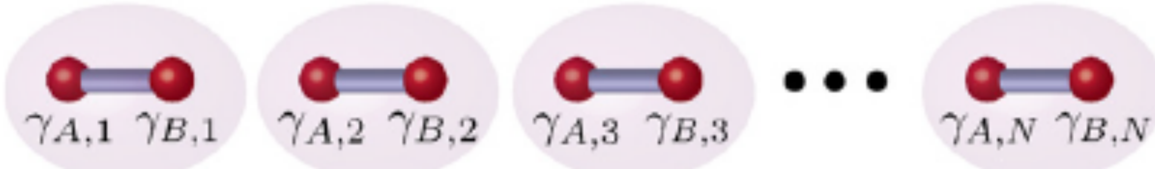
$c_j^\dagger \neq c_j$ $\rightarrow$ An electron and a hole are independent excitations.

Superconductors rescue this bad situation!

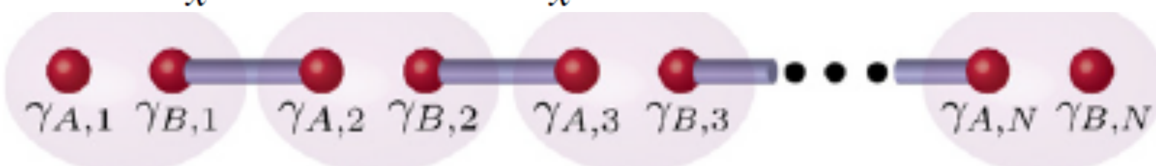


The hunt for Majorana Fermions in condensed matter

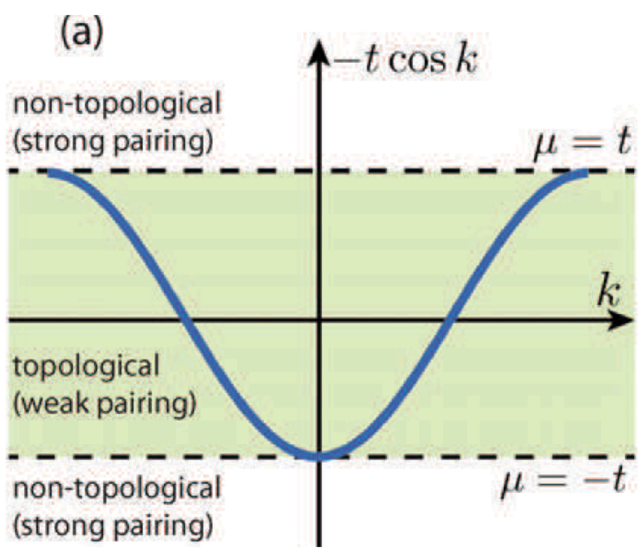
Topological superconductor: Kitaev model (p wave+hopping)



$$H = -\mu \sum_x c_x^\dagger c_x - \frac{1}{2} \sum_x (t c_x^\dagger c_{x+1} + \Delta e^{i\phi} c_x c_{x+1} + \text{H.c.})$$



t=Δ μ=0



$$\gamma_1 \equiv \gamma_{A,1} \quad \gamma_2 \equiv \gamma_{B,N}$$

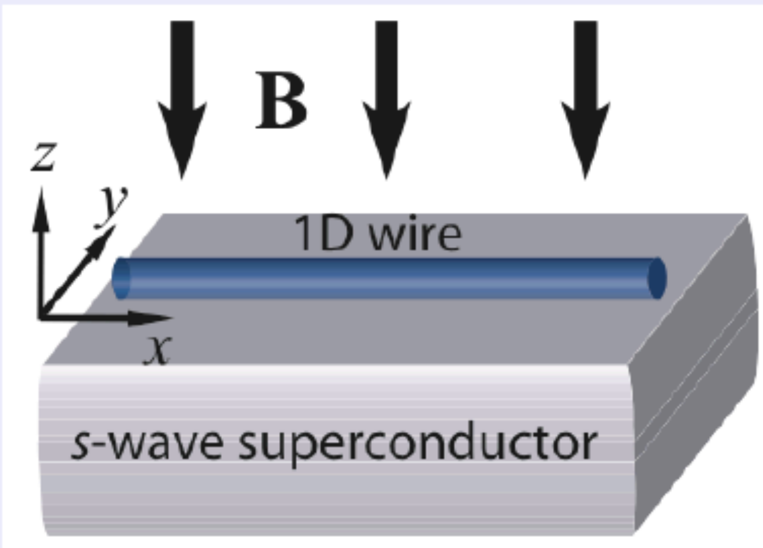
$$f = \frac{1}{2}(\gamma_1 + i\gamma_2)$$

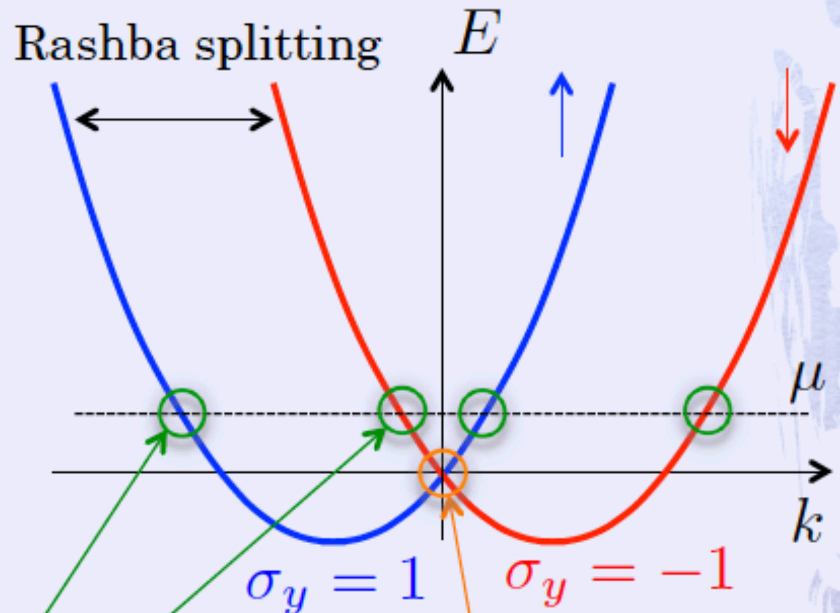
delocalized fermion

Alicea et al. Nature Physics 2011

Generate topo phase with BCS proximity, Rashba + Zeeman

Basic idea





$$H = H_{\text{wire}} + H_\Delta$$

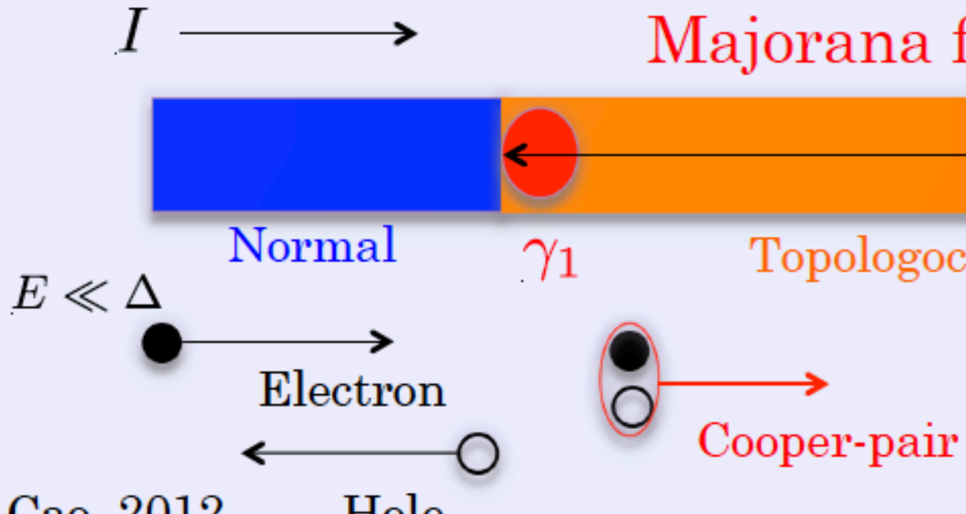
Rashba term Zeeman term

$$H_{\text{wire}} = \int dx\ \psi^\dagger \left(-\frac{\partial_x^2}{2m} - \mu \boxed{- i\alpha\sigma^y\partial_x} \boxed{+ h\sigma^z} \right) \psi$$

$$H_\Delta = \int dx\ \boxed{\Delta(\psi_\uparrow\psi_\downarrow + \psi_\downarrow^\dagger\psi_\uparrow^\dagger)}$$

Proximity effect due to *s-wave* superconductor

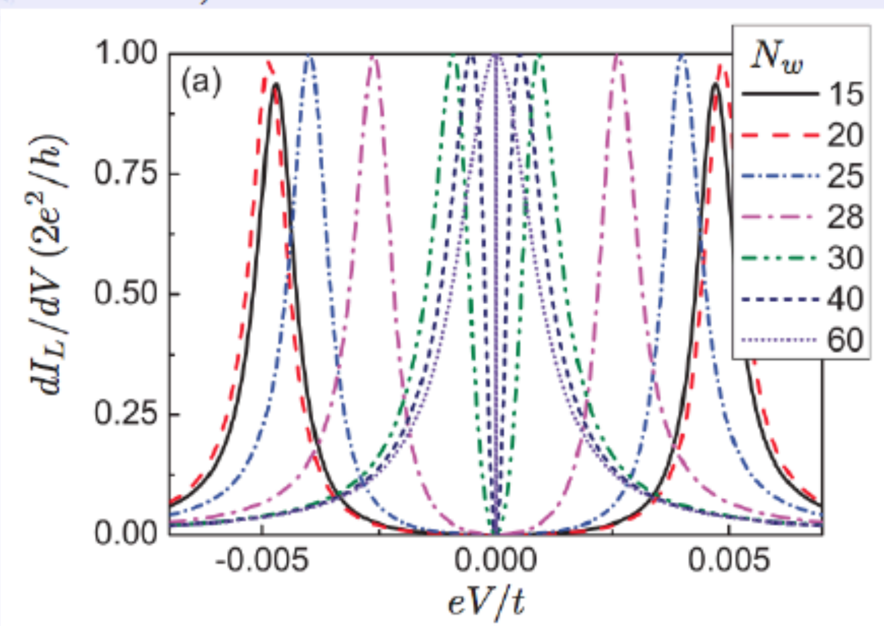
Resonant Andreev reflection

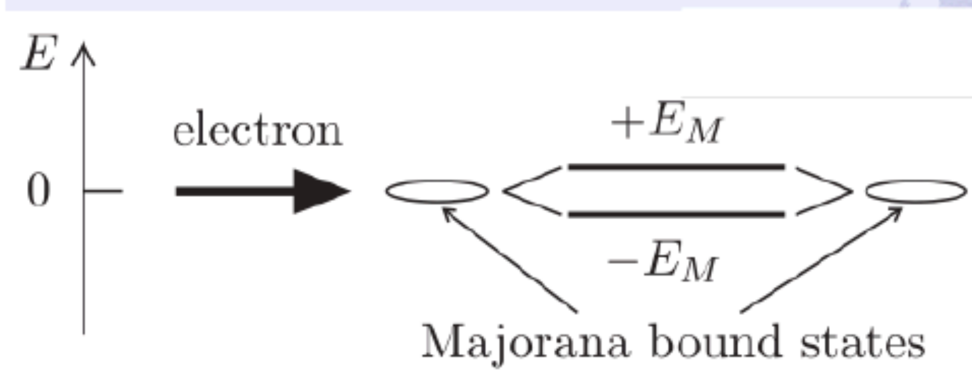


$$H_{\rm M} = iE_{\rm M}\gamma_1\gamma_2$$

$$E_{\rm M} \propto e^{-L/\xi_0}$$

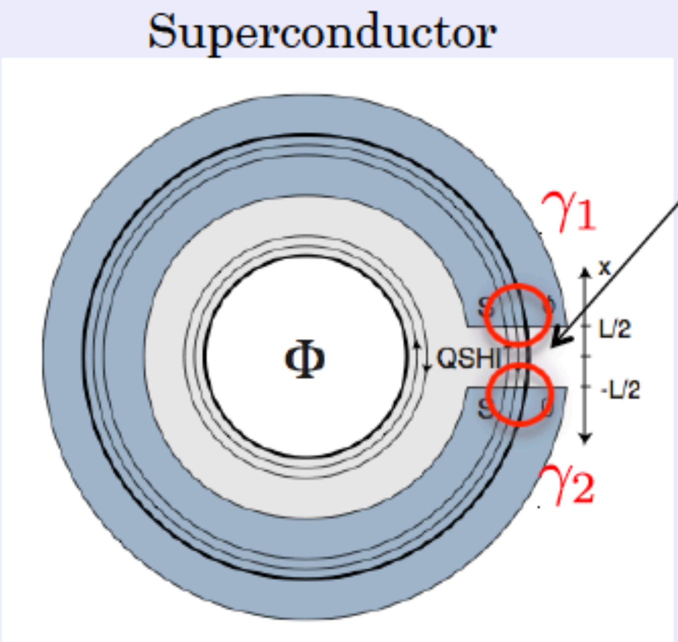
Wu-Cao, 2012





- Single resonant peak for $E_M=0$
- Peak splitting for $E_M>0$

4π periodicity



Josephson junction via Majorana

$$I(\phi) = I_0 \sin\phi/2$$

$$I(\phi + 4\pi) = I(\phi)$$

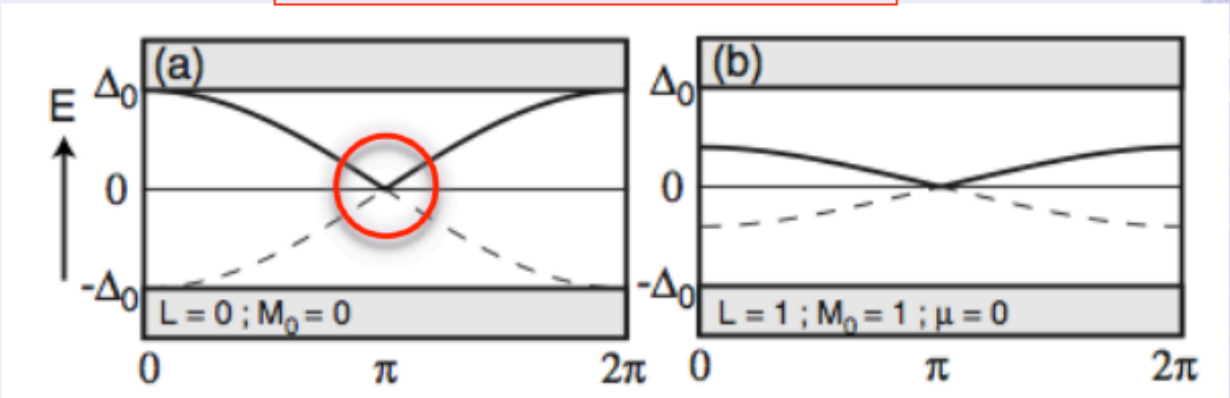
$$E_{\mathrm{M}} \simeq 0$$

No matrix element!

$$\phi \to \phi + 2\pi$$

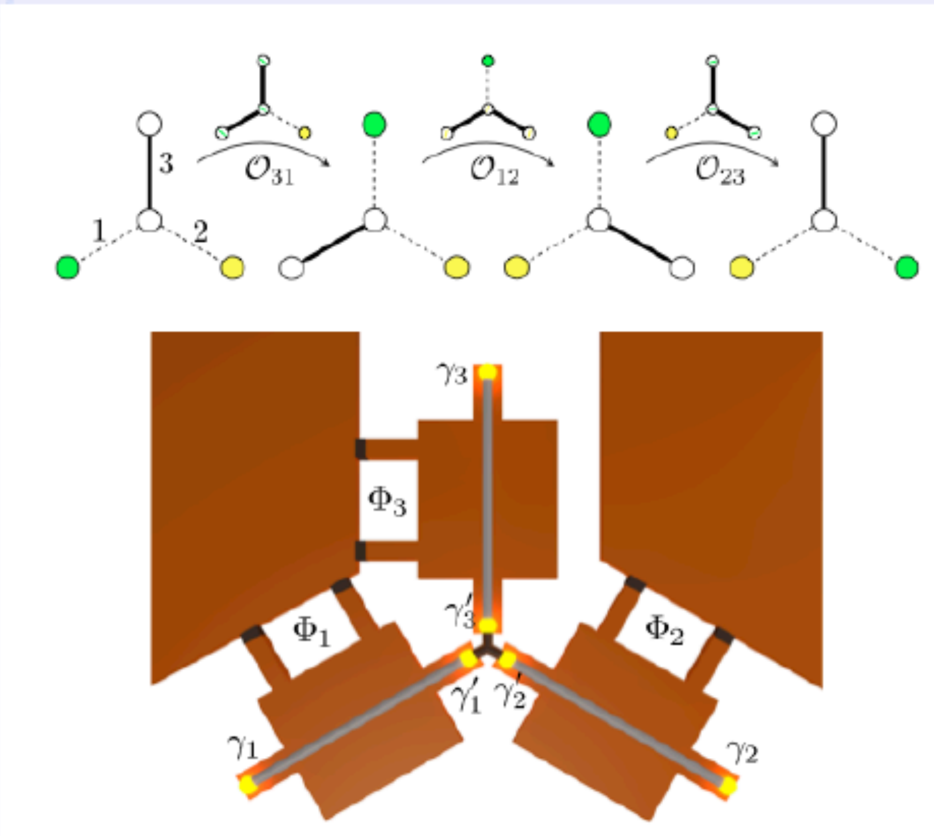
$$\gamma_1 \to \gamma_1$$

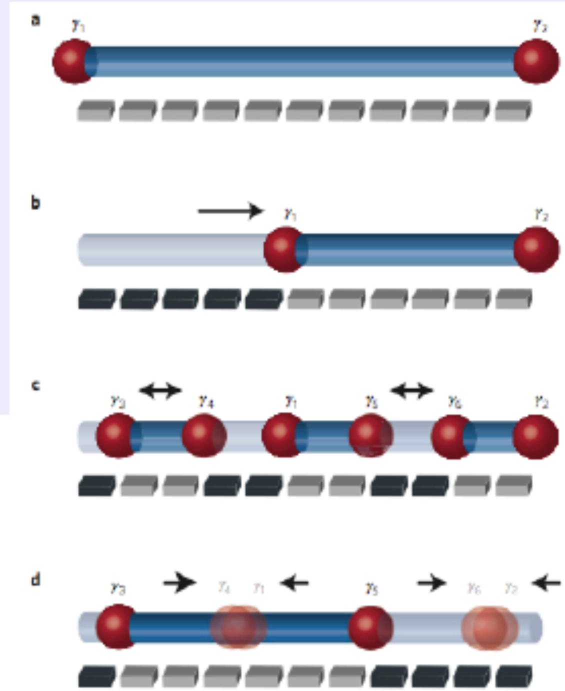
$$\gamma_2 \to -\gamma_2$$



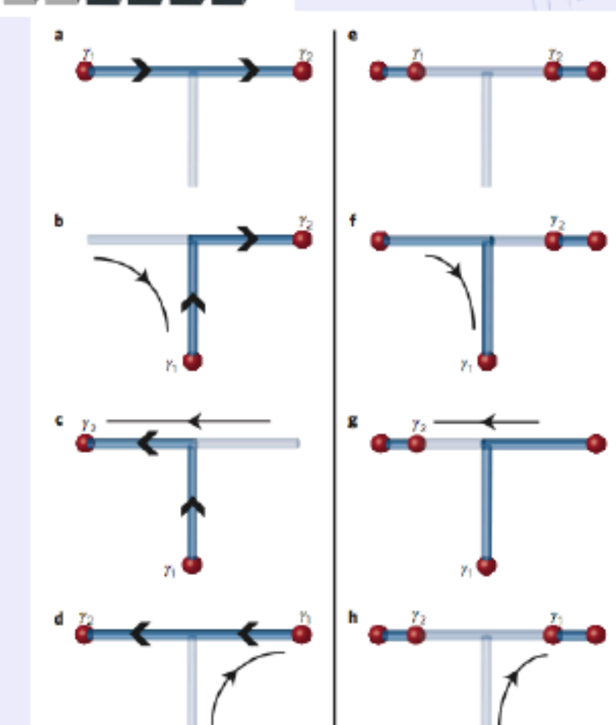
Non-Abelian statistics

van Heck 2012

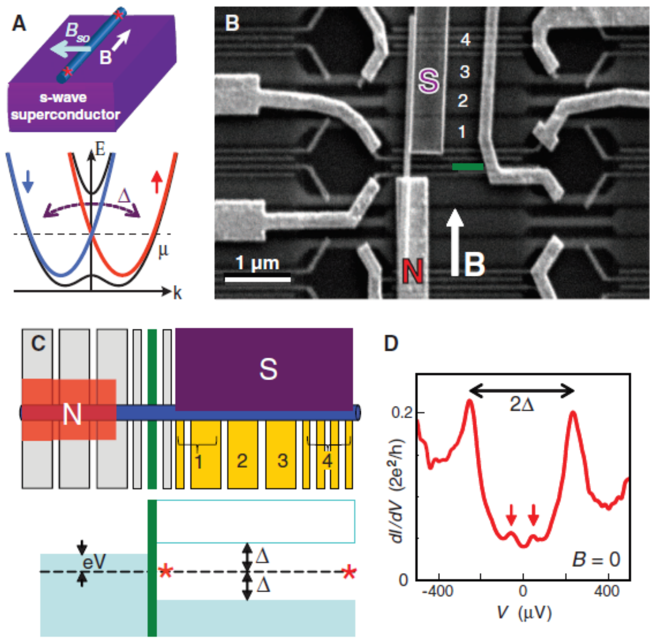


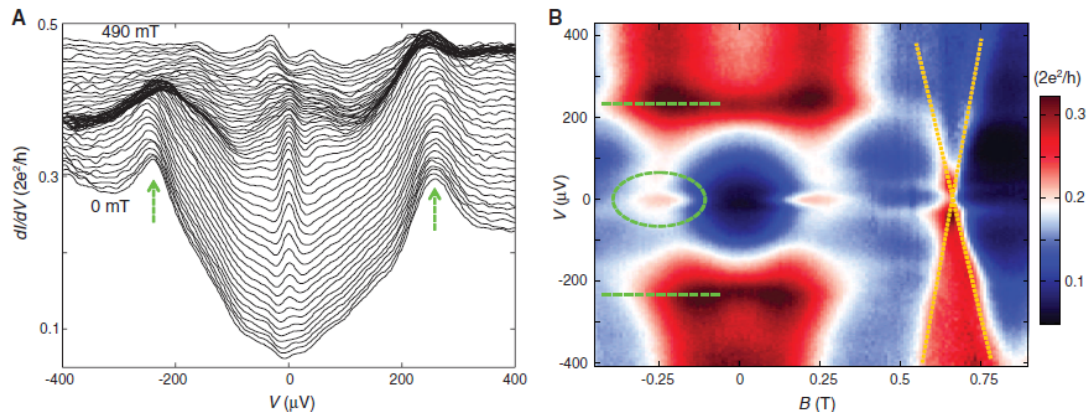


Alicea 2011



Signatures of Majorana Fermions in Hybrid Superconductor-Semiconductor Nanowire Devices
V. Mourik *et al.*
Science **336**, 1003 (2012);





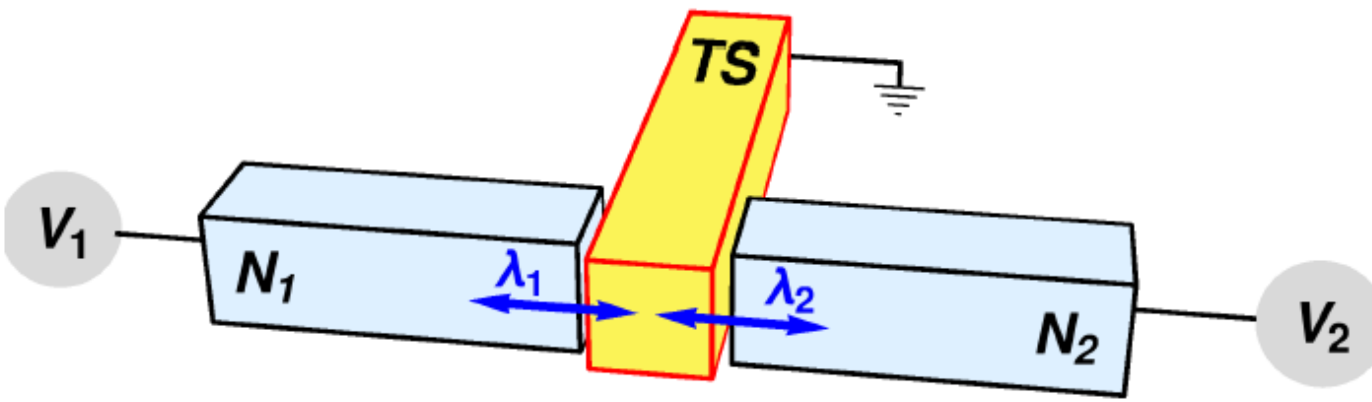
Zero bias anomaly is a potential signature of Majoranas
Is it the smoking gun ?

Several more experiments…

More proposals are needed

4) Hanbury-Brown and Twiss noise correlations in the topological superconductor beam splitter

previous work:
below gap regime, scattering theory,
shows negative (fermionic) noise correlations at equal voltages:
(Haim et al. PRB 2015)



Goal here: (continuum version of Kitaev)
(Levy-Yeyati, Zazunov, Egger PRB16)

- Treat **below** and **above** gap with **microscopic** Keldysh Green's function
- Arbitrary +/- voltages

$$H = H_{TS} + H_N + H_t$$ (TS + Normal + tunneling)

$$H_{TS} = \int_0^\infty dx \, \Psi_{TS}^\dagger(x) \left(-iv_F \partial_x \sigma_z + \Delta \sigma_y\right) \Psi_{TS}(x)$$

$$\Psi_{TS}(x) = (c_r, c_l^\dagger)^T$$

$$H_t = \frac{1}{2} \sum_{j,j'} \Psi_j^\dagger W_{jj'} \Psi_{j'}$$

Boundary Green's function

$$\check{g}_{TS}(t-t') = -i \left\langle \mathcal{T}_C \Psi(t) \Psi^\dagger(t') \right\rangle$$

$$\Psi = (c, c^\dagger)^T \qquad c = [c_l + c_r](x=0)$$

Zero temperature QFT formalism

$$G_(x,t,x',t') = -i\langle T(c_H(x,t)c_H^\dagger(x',t'))\rangle \quad H = H_0 + H_{int}$$

$$c_H(x,t) = e^{iHt}c_i(x)e^{-iHt} \qquad c_I(x,t) = e^{iH_0t}c_i(x)e^{-iH_0t}$$

Heisenberg Interaction

$$G_(x,t,x',t') = -i\langle S(-\infty,+\infty)T[c_I(x,t)c_I^\dagger(x',t')S(+\infty,-\infty)]\rangle$$

$$S(t,t') \equiv T\exp[-i\int_{t'}^{t} dt'' H_{int_I}(t'')]$$ Evolution

$$S(+\infty,-\infty)|G\rangle = e^{i\gamma}|G\rangle$$ Adiabatic switching

$$G_(x,t,x',t') = -i\frac{\langle T[c_I(x,t)c_I^\dagger(x',t')S(+\infty,-\infty)]\rangle}{\langle S(+\infty,-\infty)\rangle}$$

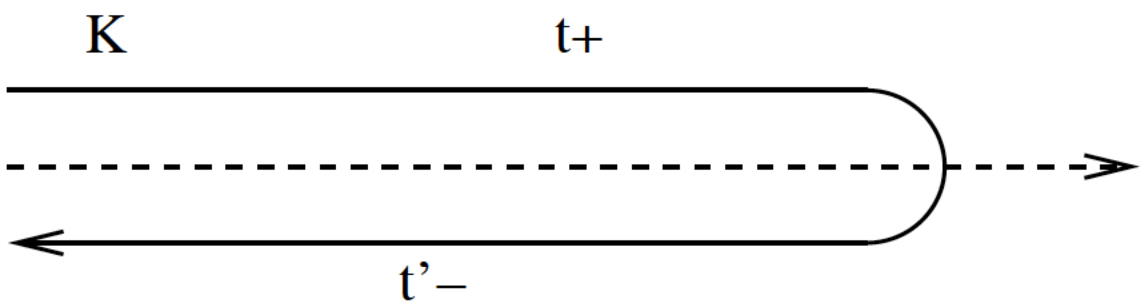
Perturbation theory, Wick's theorem, diagrams, Dyson's equation

$$G = g + g\circ\Sigma\circ G$$

Non equilibrium QFT formalism: Keldysh

$$G_(x,t,x',t') = -i\langle T[c_I(x,t)c_I^\dagger(x',t')S_K(-\infty,-\infty)]\rangle$$

$$S_K(-\infty,-\infty) \equiv T_K \exp[-i\int_{-\infty_K}^{-\infty} dt'' H_{int_I}(t'')]$$



Green's functions are now 2 by 2 matrix (times on the upper/lower contour).

Wick's theorem still works !

Dyson equation has also a matrix form

Greens functions and self energies are redundant

$$G^{+,+} + G^{-,-} = G^{+,-} + G^{-,+}$$

$$\Sigma^{+,+} + \Sigma^{-,-} = -\left(\Sigma^{+,-} + \Sigma^{-,+}\right)$$

$$\check{G} = \hat{L}\,\hat{G}\,\hat{L}^{\dagger} = \begin{pmatrix} 0 & G^{a} \\ G^{r} & G^{K} \end{pmatrix} \qquad \hat{L} = \frac{1 - i\hat{\sigma}_y}{\sqrt{2}}$$

Advanced, retarded and Keldysh Green's functions

$$G^{r}(1,1') = -i\Theta(t_1 - t_{1'})\langle\left[\hat{\psi}(1), \hat{\psi}^{\dagger}(1')\right]_{+}\rangle$$

$$G^{a}(1,1') = i\Theta(t_{1'} - t_1)\langle\left[\psi(1), \psi^{\dagger}(1')\right]_{+}\rangle$$

$$G^{K}(1,1') = -i\langle\left[\hat{\psi}(1), \hat{\psi}^{\dagger}(1')\right]_{-}\rangle = G^{+,-}(1,1') + G^{-,+}(1,1')$$

$$i\,G^{+,-}(1,1') = -\langle\hat{\psi}^{\dagger}(1')\hat{\psi}(1)\rangle \qquad i\,G^{-,+}(1,1') = \langle\hat{\psi}(1)\hat{\psi}^{\dagger}(1')\rangle$$

Dyson's equations with A, R and K

$$G^a = g^a + g^a \Sigma^a G^a$$

$$G^r = g^r + g^r \Sigma^r G^r$$

$$G^K = g^K + g^K \Sigma^a G^a + g^r \Sigma^r G^K + g^r \Sigma^K G^a$$

$$G^K = (1 + G^r \Sigma^r)\, g^K\, (1 + \Sigma^a G^a) + G^r \Sigma^K G^a$$

Σ^K Vanishes for a kinetic or scalar potential

Non equilibrium QFT formalism: Keldysh-Nambu:

When dealing with superconductors, one deals with 4 by 4 Green's functions

$$G^{-+}_{jj'}(t,t') = -i\begin{pmatrix} \langle c_j(t)c^\dagger_{j'}(t')\rangle & \langle c_j(t)c_{j'}(t')\rangle \\ \langle c^\dagger_j(t)c^\dagger_{j'}(t')\rangle & \langle c^\dagger_j(t)c_{j'}(t')\rangle \end{pmatrix}$$

$$G^{+-}_{jj'}(t,t') = +i\begin{pmatrix} \langle c^\dagger_{j'}(t')c_j(t)\rangle & \langle c_{j'}(t')c_j(t)\rangle \\ \langle c^\dagger_{j'}(t')c^\dagger_j(t)\rangle & \langle c_{j'}(t')c^\dagger_j(t)\rangle \end{pmatrix}$$

Average current

$$\langle I_j(t)\rangle \propto i(\langle c_j^\dagger(t)c_{j'}(t)\rangle - \langle c_{j'}^\dagger(t)c_j(t)\rangle)$$

Written in terms of $G_{jj'}^{+-}(t,t')$ or $G_{jj'}^{K}(t,t')$

Real time noise correlator

$$S_{jj}(t,t') \propto -\langle (c_j^\dagger c_{j'} - c_{j'}^\dagger c_j)(t)(c_j^\dagger c_{j'} - c_{j'}^\dagger c_j)(t')\rangle - \langle I_j(t)\rangle\langle I_j(t')\rangle$$

Quadratic Hamiltonian: use Wick's theorem to write this in terms of products of

$$G_{jj'}^{+-}(t,t') \qquad G_{jj'}^{-+}(t,t')$$

Boundary Green's functions

Semi infinite TS

$$g_{TS}^{R/A}(\omega) = \frac{\sqrt{\Delta^2 - (\omega \pm i0^+)^2}\,\sigma_0 + \Delta\sigma_x}{\omega \pm i0^+}$$

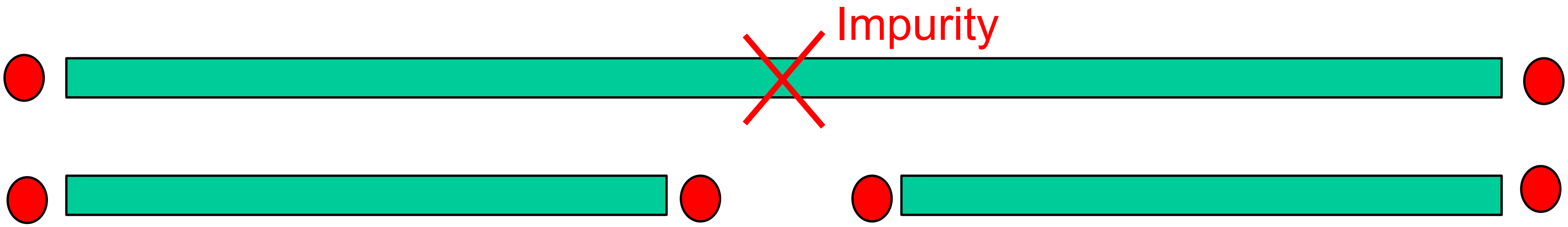
$$g_{TS}^{K}(\omega) = (1 - 2n_F(\omega))\left[g_{TS}^{R}(\omega) - g_{TS}^{A}(\omega)\right]$$

Dyson
$$G^K(\omega) = G^R(\omega)F(\omega) - F(\omega)G^A(\omega) + G^R(\omega)\left[F(\omega)W - WF(\omega)\right]G^A(\omega)$$

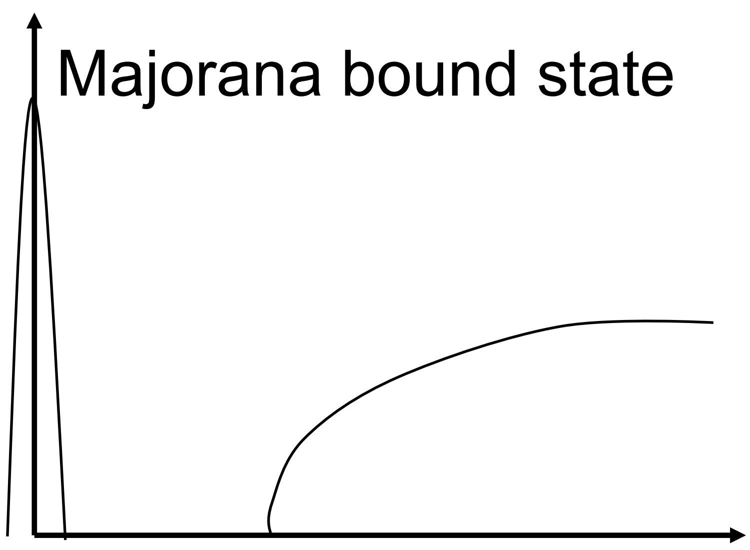
$$F_{jk}(\omega) = \delta_{jk}\left[1 - 2n_F(\omega - \mu_j\sigma_z)\right]$$

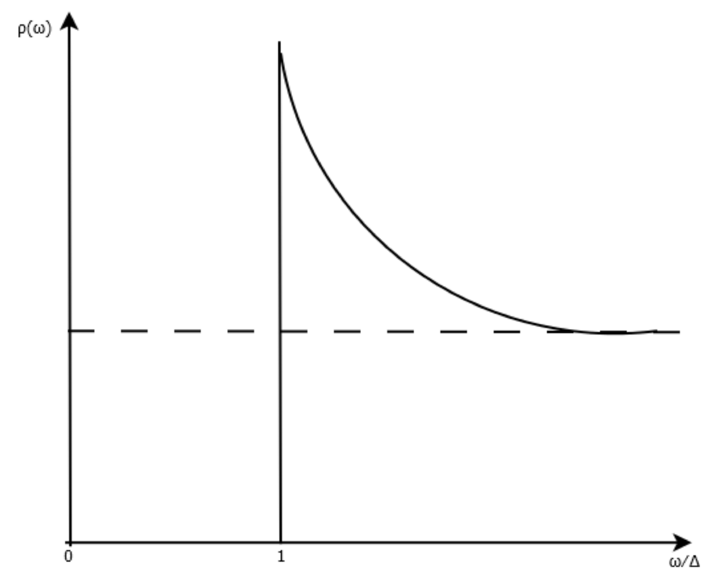
Zazunov PRB 2016

How to obtain the boundary GF ? Cut the TS wire in two !
And solve the Dyson equation



Density of states ? (TS versus BCS)





Current and noise in terms of Keldysh Green's function (Dyson solved to all orders in tunneling)

« Nozières-Cuevas» - type formulae (J. Phys C 1971, PRB 1996)

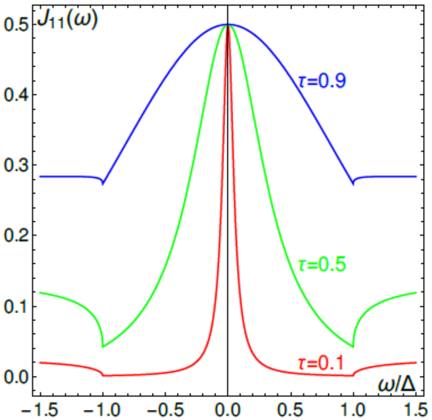
$$I_j = \frac{1}{2}\frac{e}{\hbar}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\sum_{j'\neq j}\mathrm{tr}_N\left[\sigma_z W_{jj'}G^K_{j'j}(\omega)\right]$$

$$S_{jj'} = \int_{-\infty}^{\infty} d\tau\ \left\langle \delta\hat{I}_j(\tau)\delta\hat{I}_{j'}(0)\right\rangle$$

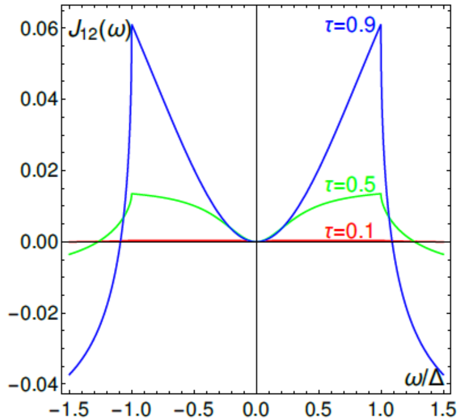
$$S_{jj'} = \int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\sum_{j_1\neq j}\sum_{j_2\neq j'}\mathrm{tr}_N\Big\{\lambda_{jj_1}\Big[G^{-+}_{j_1j_2}(\omega)\lambda_{j_2j'}G^{+-}_{j'j}(\omega) - G^{-+}_{j_1j'}(\omega)\lambda_{j'j_2}G^{+-}_{j_2j}(\omega)\Big]\Big\},$$

From Wick's theorem

Current and differential conductance



Local



Non-local

$$\tau = 4\Lambda^2/(1+\Lambda^2)^2$$

transparency

$$\Lambda = \sqrt{\lambda_1^2 + \lambda_2^2}$$

Coupling to TS

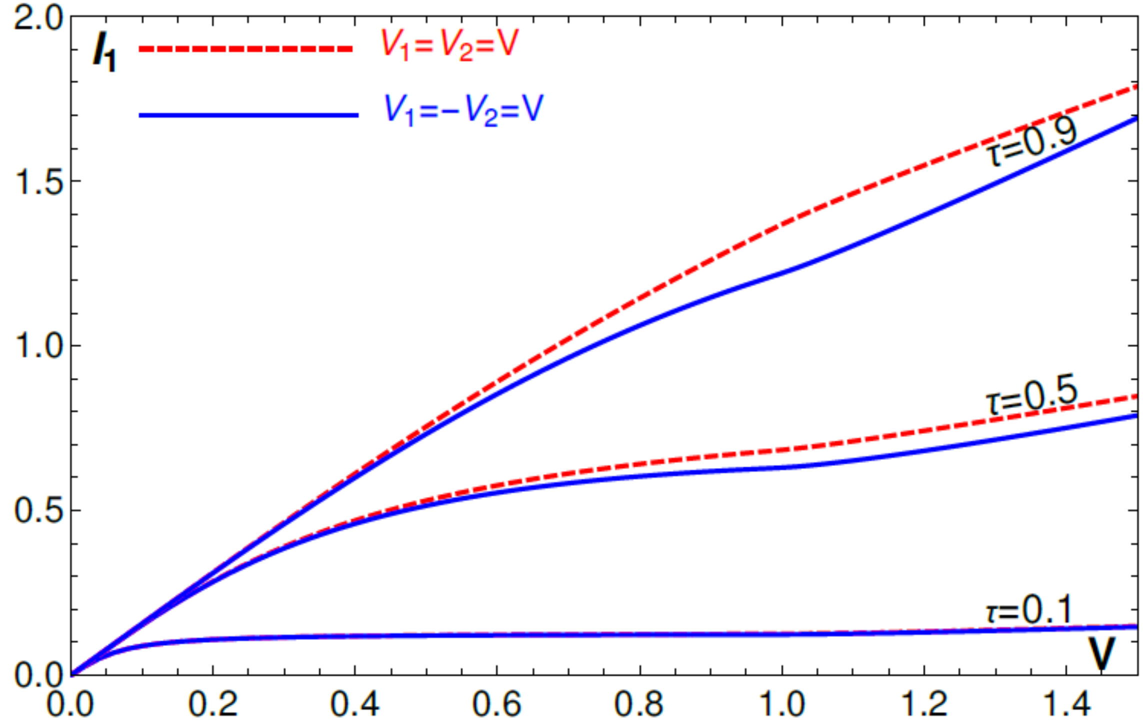
$$I_j = \frac{e}{h}\int_{-\infty}^{\infty} d\omega \sum_{k=1,2}\sum_{s=\pm} s\, n_F(\omega - s\, eV_k) J_{jk}(\omega)$$

Landauer formula

$$J_{11}(\omega) = -J_{12}(\omega) + \frac{4\lambda_1^2\Lambda^2}{(1-\Lambda^4)^2\frac{\omega^2}{\Delta^2} + 4\Lambda^4}$$

$$J_{12}(\omega) = \frac{2\lambda_1^2\lambda_2^2\left(1-\Lambda^4\right)\frac{\omega^2}{\Delta^2}}{(1-\Lambda^4)^2\frac{\omega^2}{\Delta^2} + 4\Lambda^4},$$

Non-local differential conductance (measurable experimentally ?)



For equal or opposite voltages, the current in 1 depends weakly on the symmetric/antisymmetric voltage configuration.

Differences occur at high transparency

Hanbury-Brown and Twiss noise crossed correlations

•equal voltages

$$S_{12}(V_1 = V_2 = V) = -\frac{2e^2}{h}\frac{\Gamma^2}{4}\frac{|eV|}{(eV)^2 + \Gamma^2}$$

$$\Gamma = 2\Delta\Lambda^2/(1-\Lambda^4)$$

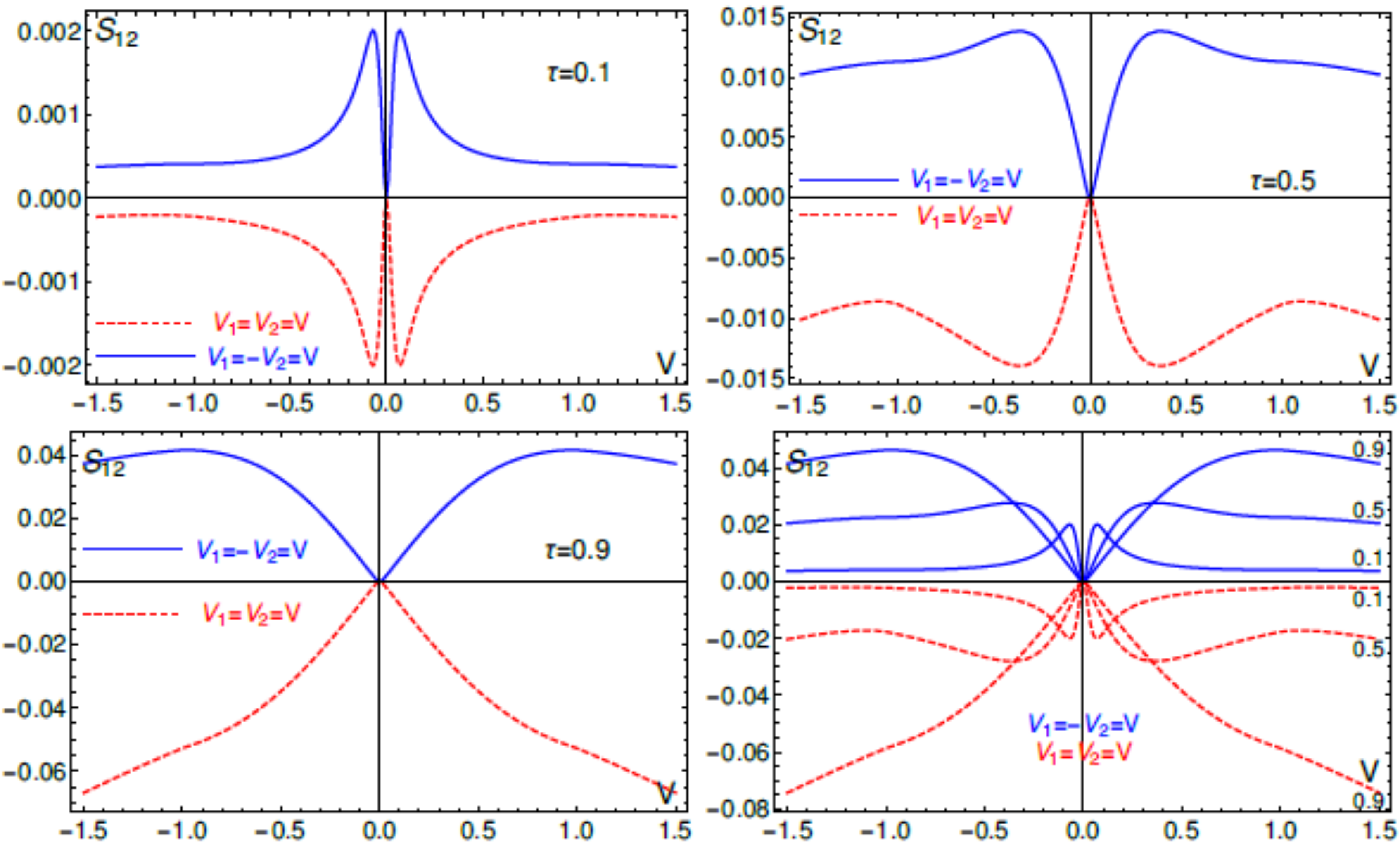
negative noise correlations like all fermionic system

•opposite voltages
positive noise
crossed correlations

$$S_{12}(V_1 = -V_2 = V) = \frac{2e^2}{h}\frac{\Gamma^2}{4}\Big[\frac{|eV|}{(eV)^2 + \Gamma^2} + \frac{2\Gamma^2 + (eV)^2}{(eV)^2 + \Gamma^2}\frac{|eV|}{\Delta^2} - \frac{2\Gamma}{\Delta^2}\tan^{-1}\left(\frac{|eV|}{\Gamma}\right)\Big]$$

•General relation bewteen auto and crossed correlations
(Martin Landauer PRB 1992)

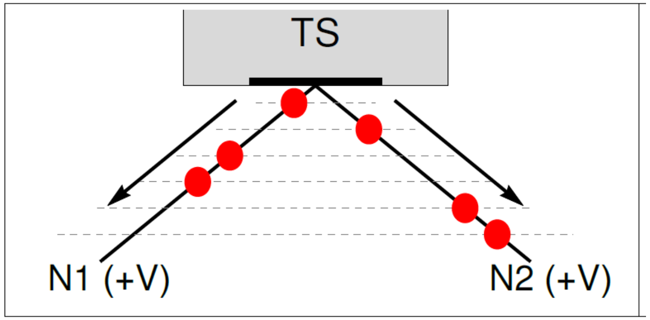
$$S_{00} = S_{11} + S_{22} + 2\,S_{12}$$



Noise crossed correlations are negative for equal voltages (normal metal « like ») and positive for opposite voltages (BCS « like »). But the origin is fundamentally different (Majorana fermion).

Physical interpretation: equal voltages

(low voltage behavior)



$$S_{00} = 2\Gamma \left[\tan^{-1}\left(\frac{|eV|}{\Gamma}\right) - \frac{|eV|/\Gamma}{1+(eV/\Gamma)^2} \right] \simeq 0$$

$$S_{11} = \Gamma \tan^{-1}\left(\frac{|eV|}{\Gamma}\right) - \frac{1}{2}\frac{|V|}{1+(eV/\Gamma)^2} \simeq \frac{|eV|}{2}$$

$$S_{12} = -\frac{1}{2}\frac{|eV|}{1+(eV/\Gamma)^2} \simeq -\frac{|eV|}{2}$$

$$\Gamma = 2\Delta\Lambda^2/(1-\Lambda^4)$$

Injection current is noiseless due to a zero bias resonance (ideal transmission)

$$I_0 = 2(e^2/h)V$$

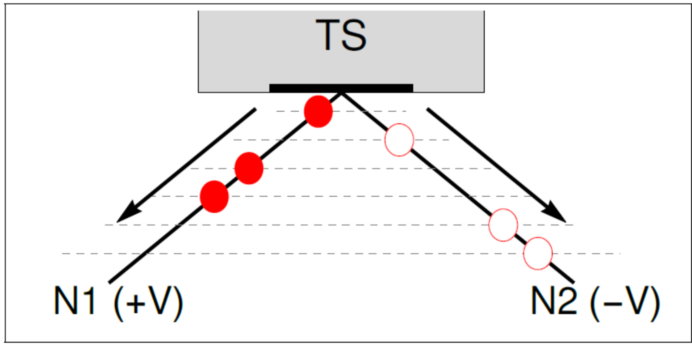
The injection current
Is partitioned in 1 an 2
The normal leads noise is

$$S_{jj} \equiv eI_j(1-T) = \frac{e^2}{h}\frac{|eV|}{2}$$

$$S_{12} = -S_{11}$$

Crossed correlations
(all fermionic behavior) → negative crossed correlations

Interpretation: opposite voltages



$$S_{00} = 2\Gamma \tan^{-1}\left(\frac{|eV|}{\Gamma}\right) \simeq 2|eV|$$

$$S_{11} = \Gamma \tan^{-1}\left(\frac{|eV|}{\Gamma}\right) - \frac{|eV|/2}{1+(eV/\Gamma)^2} - f(V,\Gamma) \simeq \frac{|eV|}{2}$$

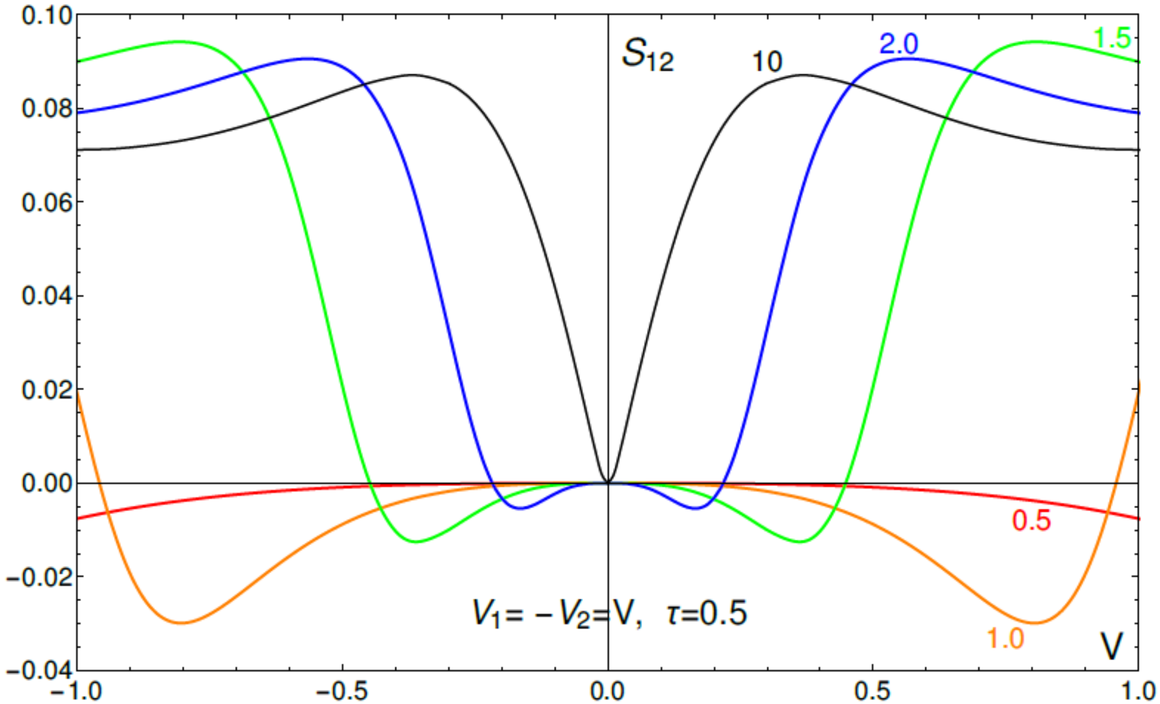
$$S_{12} = \frac{1}{2}\frac{|eV|}{1+(eV/\Gamma)^2} + f(V,\Gamma) \simeq \frac{|eV|}{2}$$

Same ingredients: coupling to Majorana is e-h symmetric
1 collects e, 2 collects h
→ TS particle current is noiseless, TS charge current is noise-full
e-h partitioning leads to $I_1 = -I_2 = (e^2/h)V$

Auto-correlation noise $S_{11} = S_{22} = (e^3/h)|V|/2$
Crossed correlation noise are positive, as carriers bear opposite charge.

TS lead noise is thus $S_{00} = 2(e^3/h)|V|$

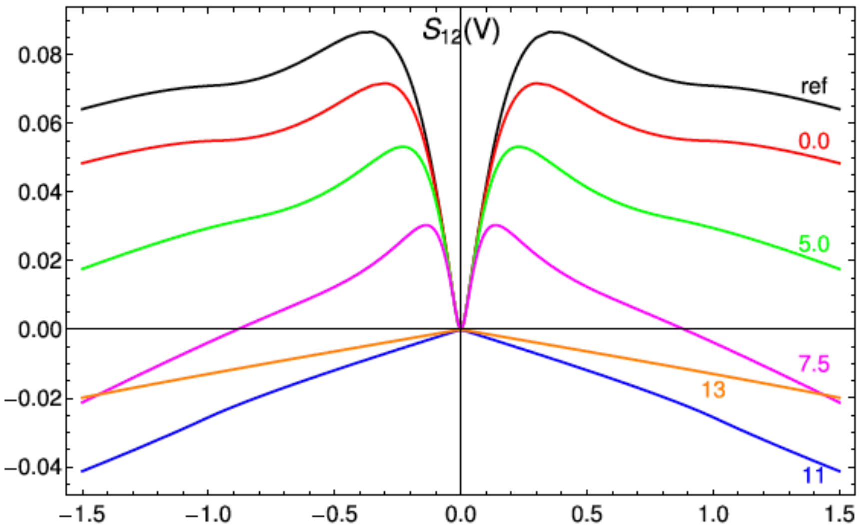
Extension 1: Finite length TS wire, **opposite voltages**
(the two Majorana's « communicate » for small wire length L)



L in units of ξ
(TS coh. length)

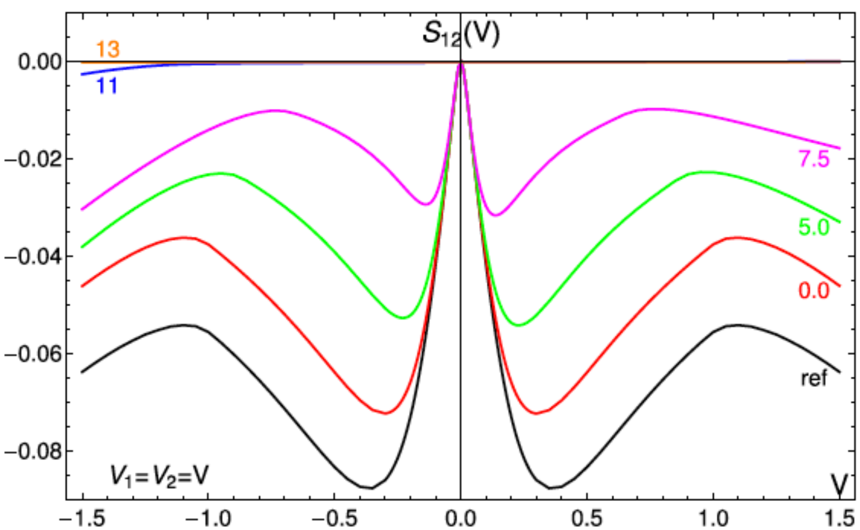
→Reversal of sign of noise crossed correlations for small TS wire length, because the two Majorana hybridize.

Varying the intrinsic chemical potential of the TS wire (allows to drive the TS to a topologically trivial phase)



opposite voltages

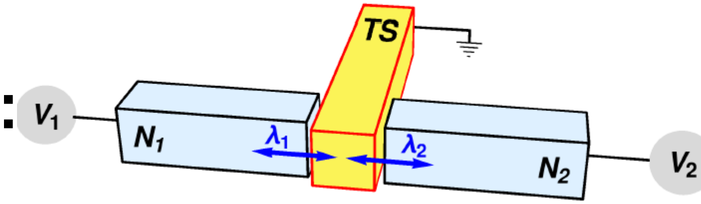
reversal of the sign of crossed correlation



equal voltages

asymmetry develops when topologically trivial phase is reached

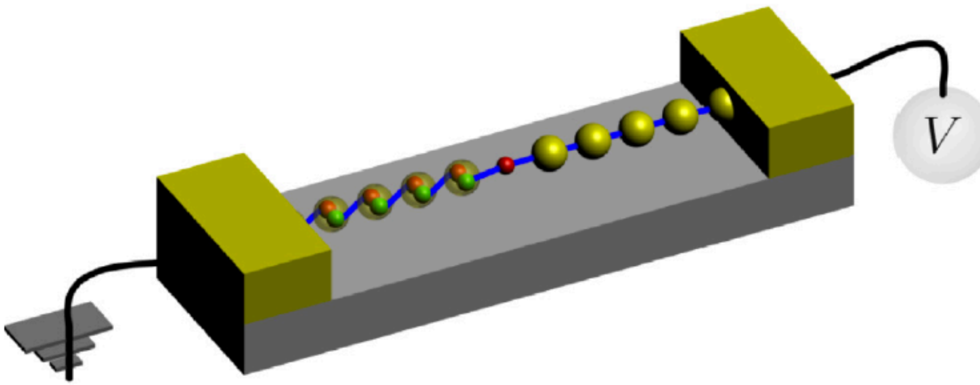
CONCLUSIONS ON TOPO SPLITTER:

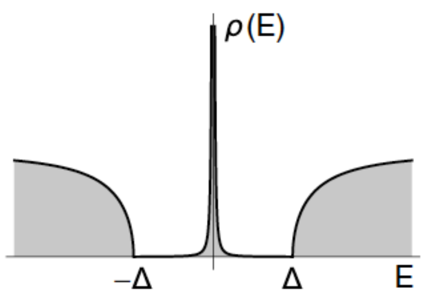


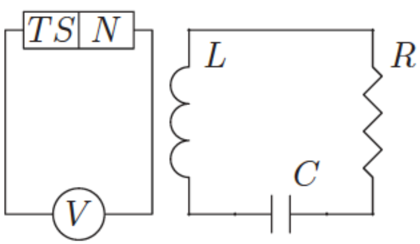
- Keldysh Green's function approach to hybrid N-TS-N systems, treat: below/above gap, finite TS, doping of TS
- Non local differential conductance
- Crossed correlations < 0 (fermionic) at equal voltages
- Crossed correlations > 0 at opposite voltages: Majorana converts electrons into holes.
- Reversal of noise crossed correlations (opposite V) when 2 Majoranas overlap.
- Transition of noise crossed correlations when driving to topologically trivial phase.

NSN Beam splitter: PRB 83, 125421 (2011); PRB 85, 035419 (2012)
TS Beam splitter : arXiv:1611.03776, Phys Rev B 95, 054514 (2017)

5) Finite frequency noise in a normal metal–topological superconductor junction

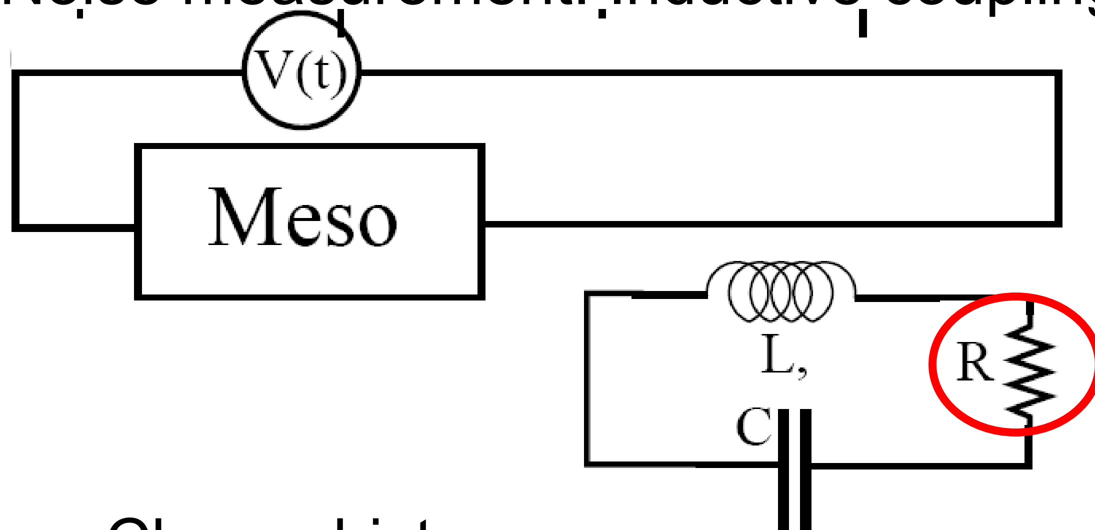






Introduction to finite frequency noise

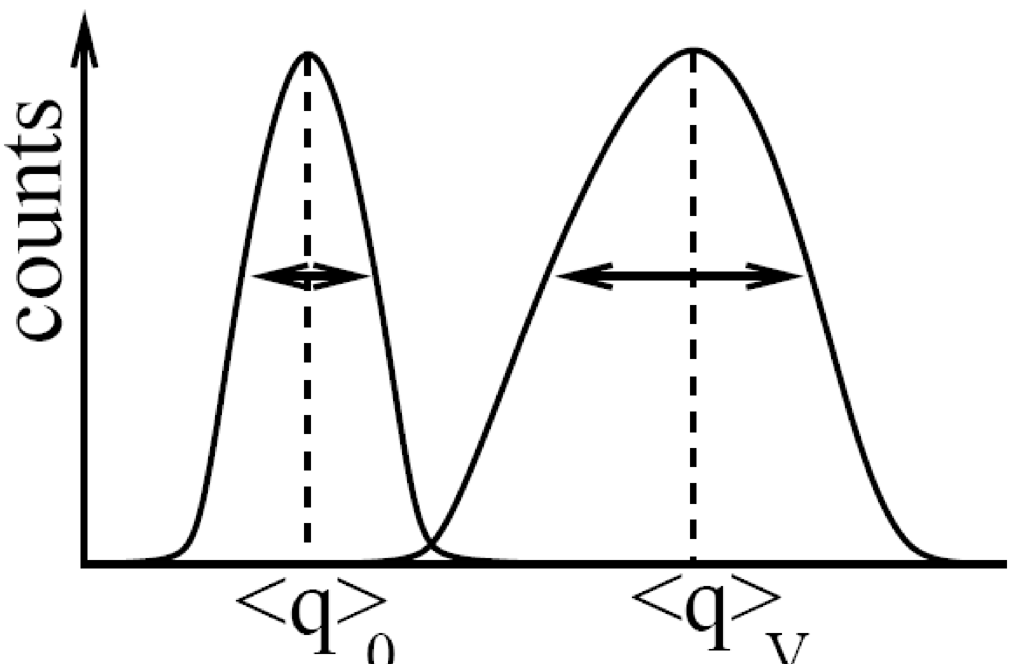
Noise measurement: Inductive coupling Lesovik JETP97, Gavish…PRB2000



$$L_0 = \frac{M\dot{x}^2}{2} - \frac{Dx^2}{2} - \alpha \dot{I} x$$

NOW
With damping

Charge histogram



Repeated measurement of the charge on the plate of the capacitor:

measure excess width which correspond to finite frequency noise

For a finite frequency measurement, the device and the detector have to be treated on the same footing

→Two unsymmetrized noise correlators:

$$S_+(\Omega) = \int dt \langle I(0) I(t) \rangle \exp(i\Omega t)$$

emission to the measuring circuit

$$S_-(\Omega) = \int dt \langle I(t) I(0) \rangle \exp(i\Omega t)$$

absorption from the mesoscopic device

Measured noise (from charge fluctuations on the capacitor)

is a combination of emission and absorption term.

$$\langle x^2(0) \rangle = \frac{\pi \alpha}{\eta (2M)^2} \left[(N(\Omega) + 1) S^+(\Omega) - N(\Omega) S^-(\Omega) \right]$$

x charge on capacitor, η adiabatic parameter

Measured noise diverges with $\eta \to 0$!!!

- What is the physical origin of η ?

« old » literature: Radiation Line width for Josephson effect (Larkin+ Ovchinikov, JETP 60's) du to EM environment

- For noise measurement, add dissipation in LC, modeled by a bath of oscillators.

- Use Keldysh approach (bath+ LC decoupled at t=-infinity)

In order to resolve the divergence problem

Result for fluctuations:

$$\delta\langle q^2\rangle = 2\alpha^2 \int_0^\infty \frac{d\omega}{2\pi}\omega^2[\chi''(\omega)]^2 \times \left(S_+(\omega) + N(\omega)(S_+(\omega) - S_-(\omega))\right)$$

Noise correlators

$$S_+(\omega) = \int dt \langle I(0)I(t)\rangle e^{i\omega t} \qquad S_-(\omega) = S_+(-\omega)$$

Generalized susceptibility

$$\chi''(\omega) = J(|\omega|)/[M^2(\omega^2 - \Omega^2)^2 + J^2(|\omega|)$$

Bath spectral function

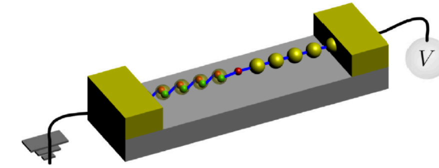
$$J(\omega) = \pi \sum_n \lambda_n^2/(2M_n\Omega_n)\delta(\omega - \Omega_n) \qquad \sim \omega$$

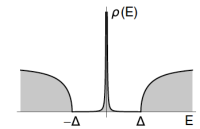
Ohmic

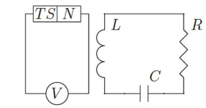
$N(\omega)$ Bose Einstein distribution

Square of a Lorentzian with zero damping ! → fluctuations diverge

Back to NTS junction:
Real time noise correlator







$$S_{jl}(t,t') = \langle I_j(t) I_l(t') \rangle - \langle I_j(t) \rangle \langle I_l(t') \rangle$$

$$S_{11}(t,t') = \lambda^2 e^2 \mathrm{Tr}_{\mathrm{N}}[G_{00}^{-+}(t,t') G_{11}^{+-}(t',t) - G_{01}^{-+}(t,t') G_{01}^{+-}(t',t)].$$

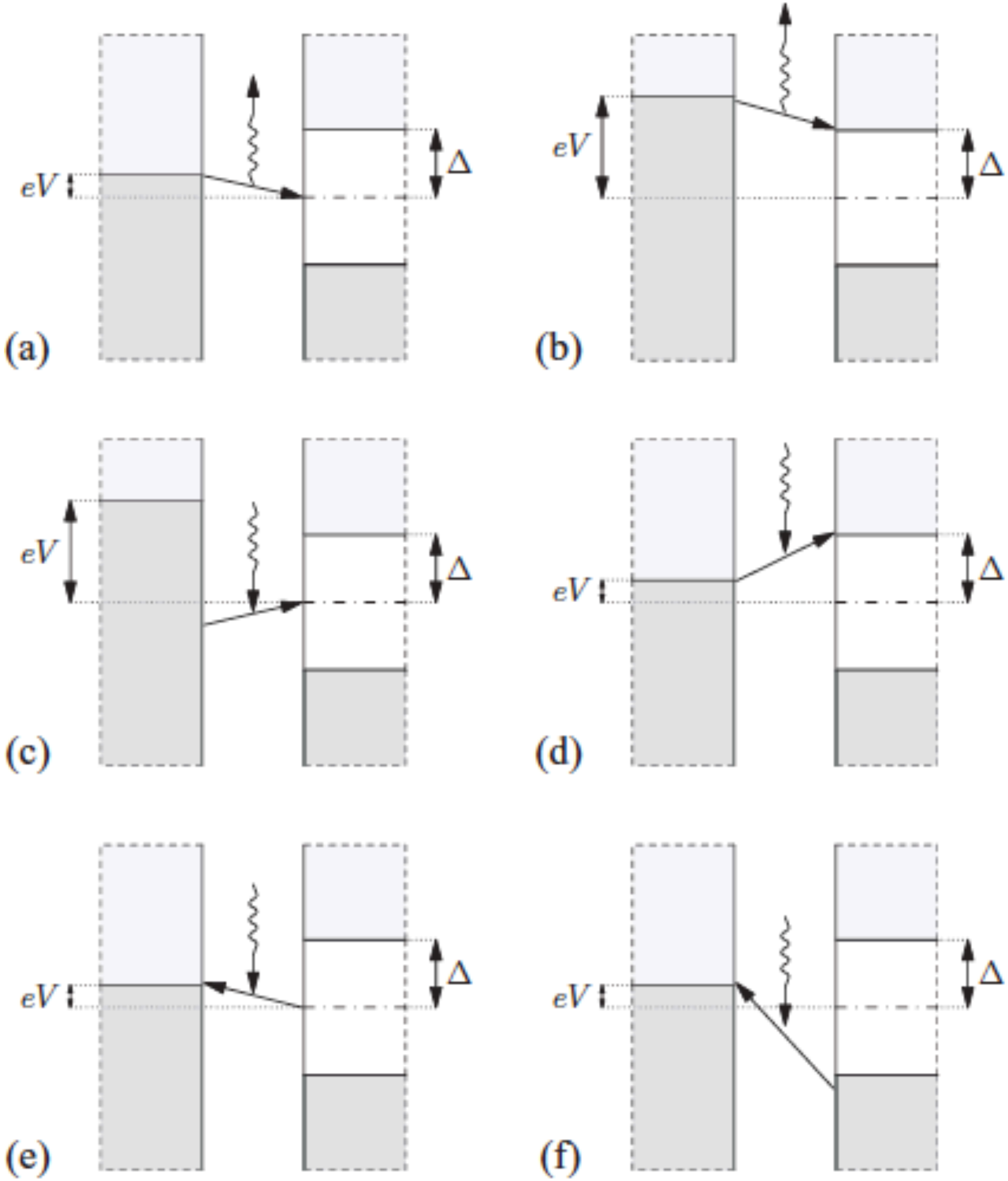
Emission and absorption noise

$$S^+(\Omega) = \int_{-\infty}^{+\infty} dt \, \langle \delta I(0) \delta I(t) \rangle \, \mathrm{e}^{i\Omega t}$$

$$S^+(\Omega) = S^-(-\Omega)$$

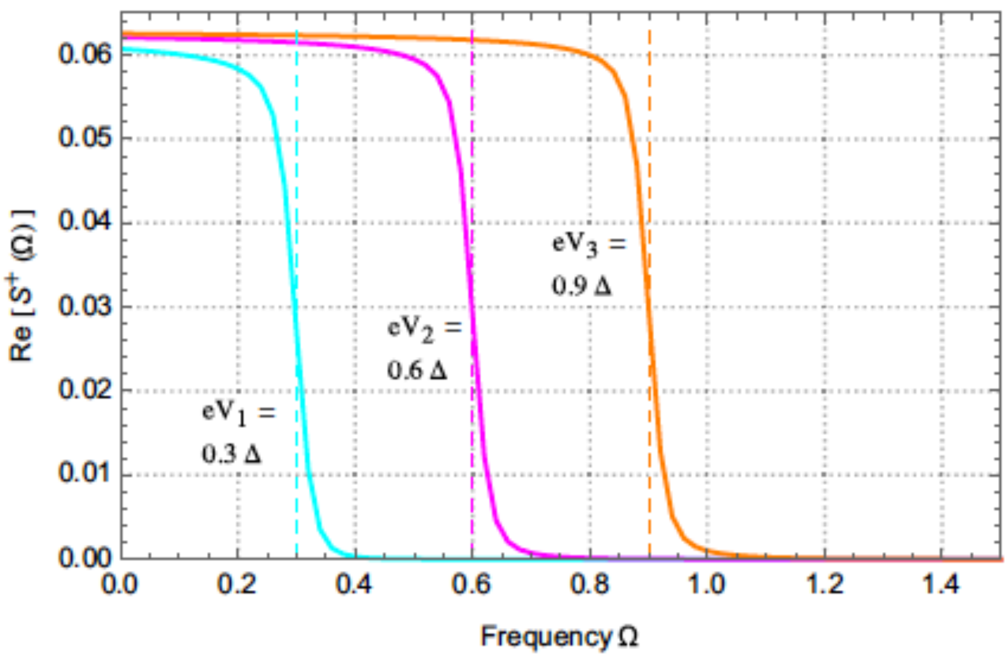
$$S^-(\Omega) = \int_{-\infty}^{+\infty} dt \, \langle \delta I(t) \delta I(0) \rangle \, \mathrm{e}^{i\Omega t}$$

$$S^+(\Omega) = \lambda^2 e^2 \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} \mathrm{Tr}_{\mathrm{N}}[G_{00}^{-+}(\omega) G_{11}^{+-}(\omega+\Omega) - G_{01}^{-+}(\omega) G_{01}^{+-}(\omega+\Omega)],$$



Energy diagrams of processes which contribute to finite frequency Emission and Absorption noise:

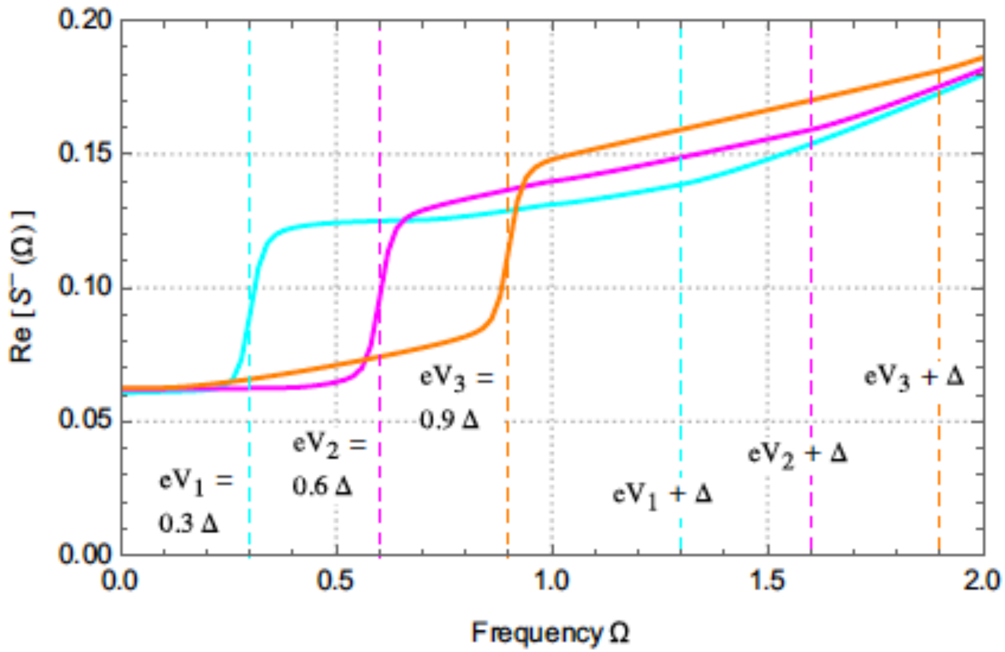
In addition to empty States, the role of the Majorana fermion is crucial



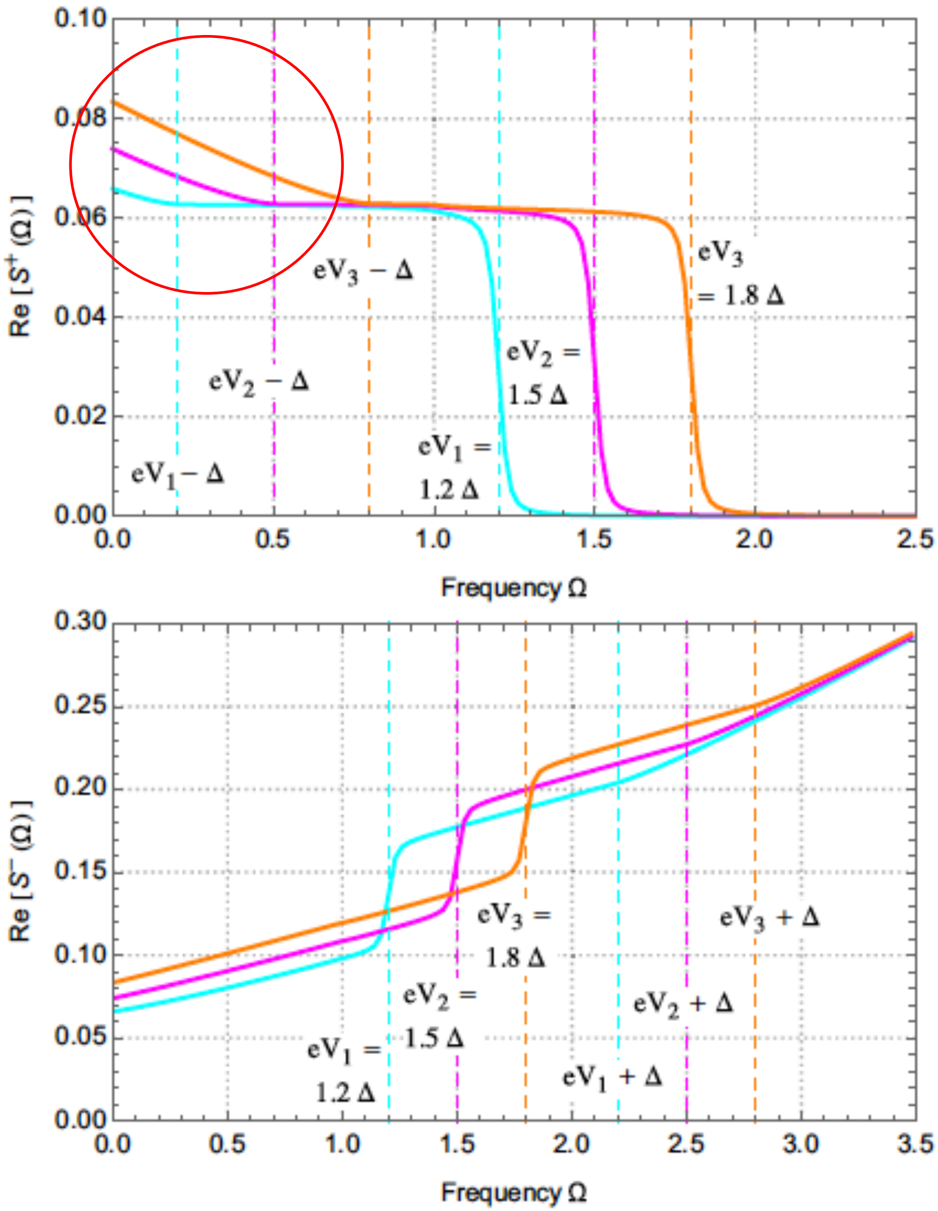
Subgap regime: low transparency

Noise plateau $[0, eV]$ and sharp drop beyond

Emission

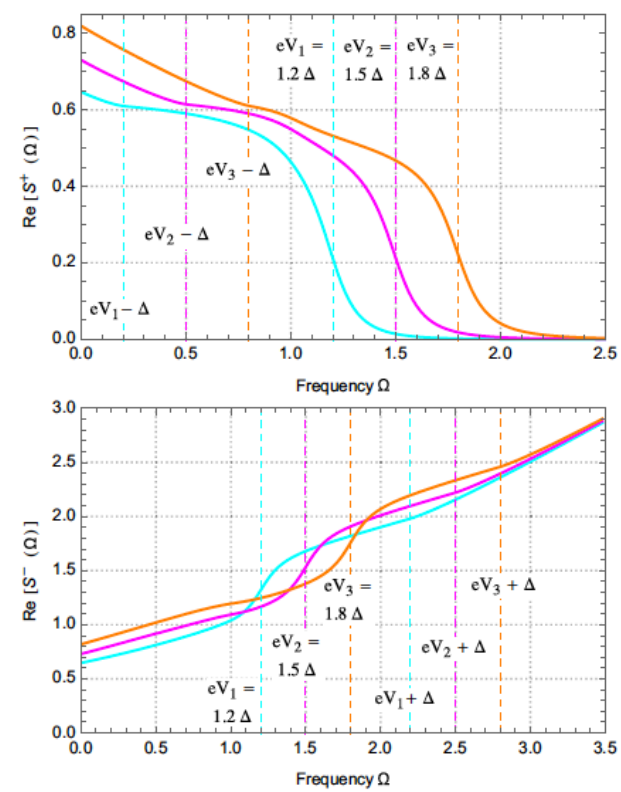


Absorption



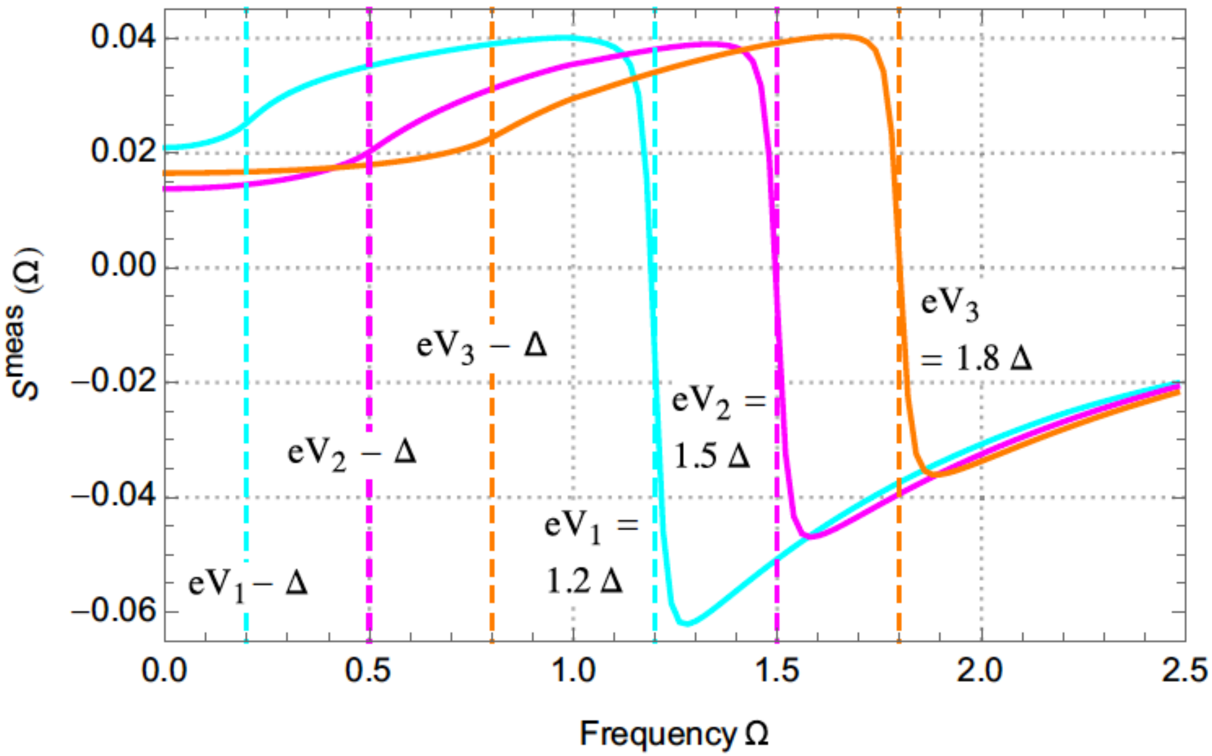
Above gap regime:
(low tranparency)
Noise plateau
Extends from
$[eV\text{-}\Omega, eV]$

(high transparency spoils)



Measurable noise: at low temperature, it is identical to $S^+(\Omega)$

But when the temperature of the detector is comparable to the gap, the plateau is distorted



$$S_{\mathrm{meas}}(\Omega) = K\{S^+(\Omega) + N(\Omega)[S^+(\Omega) - S^-(\Omega)]\}$$

Can this noise signature be reproduced by a non topological system bearing zero energy Andreev bound states ?

Perform the noise calculation of an N- dot – S (BCS) system using a Keldysh path integral method.

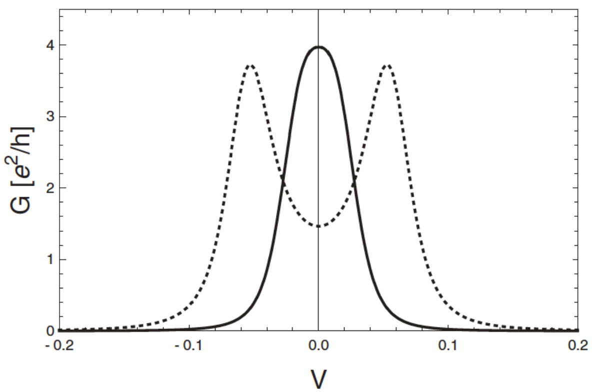
$$H = H_D + \sum_{j=S,N} H_j + H_T \quad H_D = \epsilon \sum_{\sigma=\uparrow,\downarrow} d_\sigma^\dagger d_\sigma$$

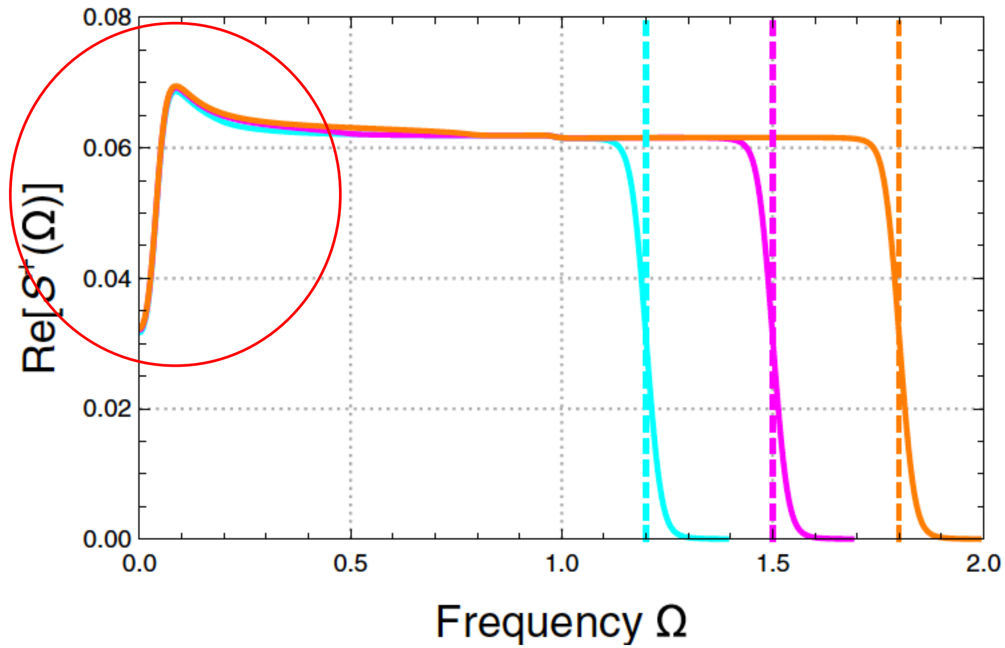
$$H_j = \sum_k \Psi_{j,k}^\dagger (\xi_k \sigma_z + \Delta_j \sigma_x) \Psi_{j,k}$$

$$\begin{aligned}
S_j(\Omega) = &-\frac{e^2}{2}\mathrm{Re}\int \frac{d\omega}{2\pi}\mathrm{Tr}_N\{\sigma_3[\Sigma_j^K \mathcal{G}^a + \Sigma_j^r \mathcal{G}^K - \Sigma_j^a \mathcal{G}^a + \Sigma_j^r \mathcal{G}^r]_\omega \sigma_3 [\Sigma_j^K \mathcal{G}^a + \Sigma_j^r \mathcal{G}^K + \Sigma_j^a \mathcal{G}^a - \Sigma_j^r \mathcal{G}^r]_{\omega+\Omega} \\
&- \sigma_3[\Sigma_j^r \mathcal{G}^r \Sigma_j^K + \Sigma_j^K \mathcal{G}^a \Sigma_j^a + \Sigma_j^r \mathcal{G}^K \Sigma_j^a - \Sigma_j^a \mathcal{G}^a \Sigma_j^a + \Sigma_j^r \mathcal{G}^r \Sigma_j^r]_\omega \sigma_3 [\mathcal{G}^K + \mathcal{G}^a - \mathcal{G}^r]_{\omega+\Omega}\} \\
&- \frac{e^2}{4}\mathrm{Re}\int \frac{d\omega}{2\pi}\mathrm{Tr}_N\{\sigma_3[\Sigma_j^a - \Sigma_j^r - \Sigma_j^K]_\omega \sigma_3 [\mathcal{G}^a - \mathcal{G}^r + \mathcal{G}^K]_{\omega+\Omega} + \sigma_3[\mathcal{G}^r - \mathcal{G}^a + \mathcal{G}^K]_\omega \sigma_3 [\Sigma_j^r - \Sigma_j^a - \Sigma_j^K]_{\omega+\Omega}\}
\end{aligned}$$

$\mathcal{G}$ Dressed dot Green’s function (chevallier PRB 2011)

N-dot-S: dot coupling between N and S have to be equal to reproduce the zero bias anomaly of the conductance.





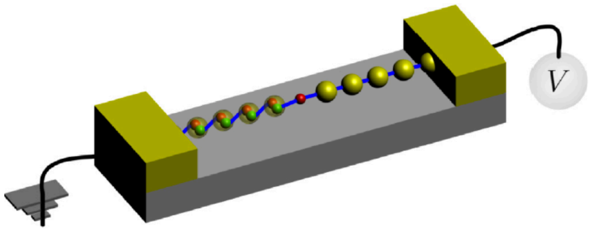
Granted, the emission noise (above gap) bears a plateau, but there is a dip close to 0 frequency associated with the Fano factor ½ of a symmetric junction.

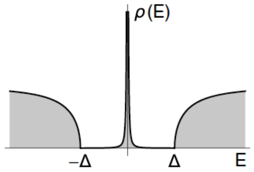
→ We rule out that the NTS junction noise signal could be attributed to a Non Topological system with zero energy Andreev bound states.

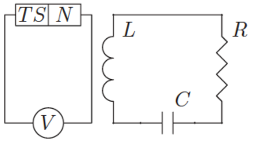
Conclusion:

The finite frequency noise of a normal metal / topological superconductor junction has uniques features associated with the Majorana bound state. Another evidence of Majorana physics.

(tunnel junctions are the best candidates for this observation)







5) Giant noise in a junction between three toplogical superconductors.

Giant Shot Noise from Majorana Zero Modes in Topological Trijunctions

Thibaut Jonckheere[1], Jérôme Rech[1], Thierry Martin[1]
Alex Zazunov[2], Reinhold Egger[2], Alfredo Levy Yeyati[3]

1 : Centre de Physique Théorique, Campus de Luminy, Marseille France
2 : Institut für Theoretische Physik, Düsseldorf, Germany
3: Condensed Matter Physics Center (IFIMAC), Universidad Autonoma de Madrid

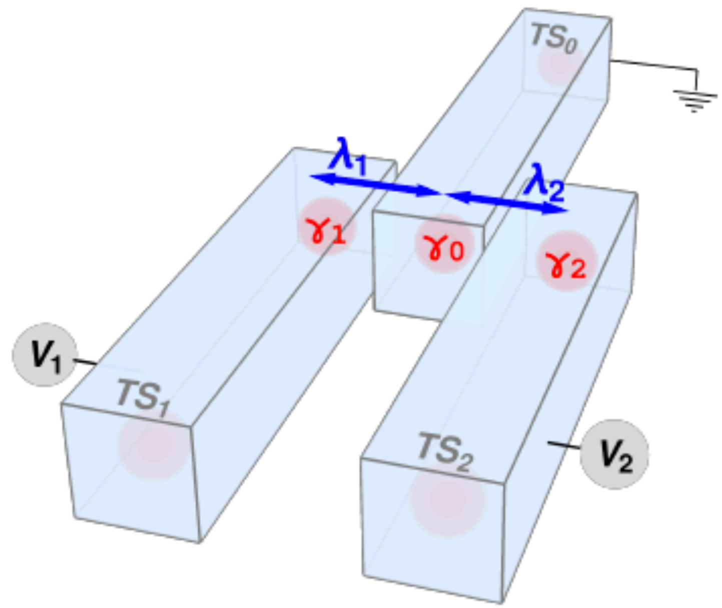
Phys. Rev. Lett. **122**, 097003 (2019)

Context - Objective

Motivations

- Study Majorana bound states in multi-terminal setup involving 3 Topological Superconductors nanowires
- Find clear signatures of Majorana bound states, more robust than the conductance peak or the 4π periodicity
- Current correlation (*noise*) provide unexpected and original signal

The TS-TS-TS setup



A grounded TS_0 nanowire is coupled to two voltage-biased TS_1 and TS_2

We consider here $V_1 = -V_2$ (similar results for $pV_1 + qV_2 = 0$)

3 coupled Majorana bound state $\Rightarrow$
one effective fermion and one effective Majorana bound state

Boundary Green functions

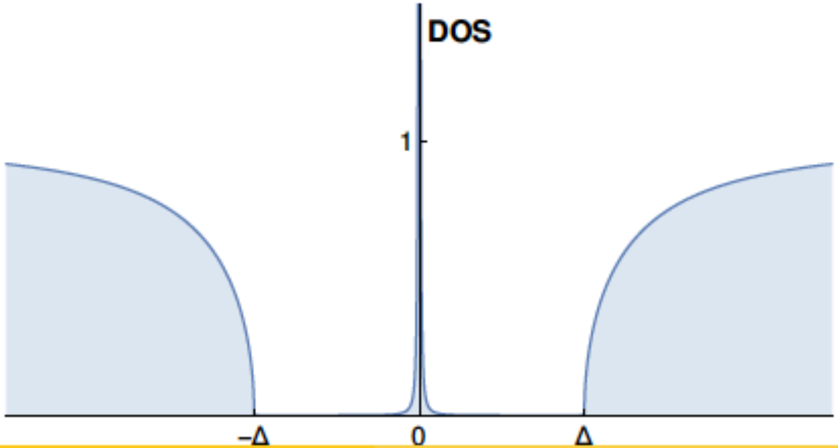
BGF for a semi infinite TS

- Transport can be described in terms of boundary Green functions coupled by tunneling processes
- Boundary GF for TS wire : $\check{g}_{TS}(t-t') = -i \langle \mathcal{T}_C \Psi(t) \Psi^\dagger(t') \rangle$,
- Explicit expression of retarded/advanced GF :

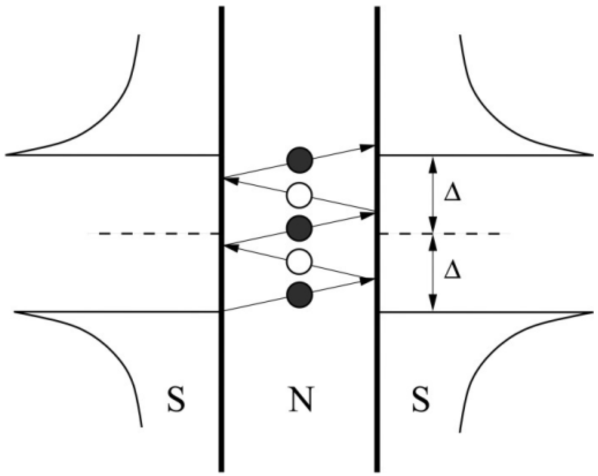
$$g_{TS}^{R/A}(\omega) = \frac{\sqrt{\Delta^2 - (\omega \pm i0^+)^2}\,\sigma_0 + \Delta \sigma_x}{\omega \pm i0^+}$$

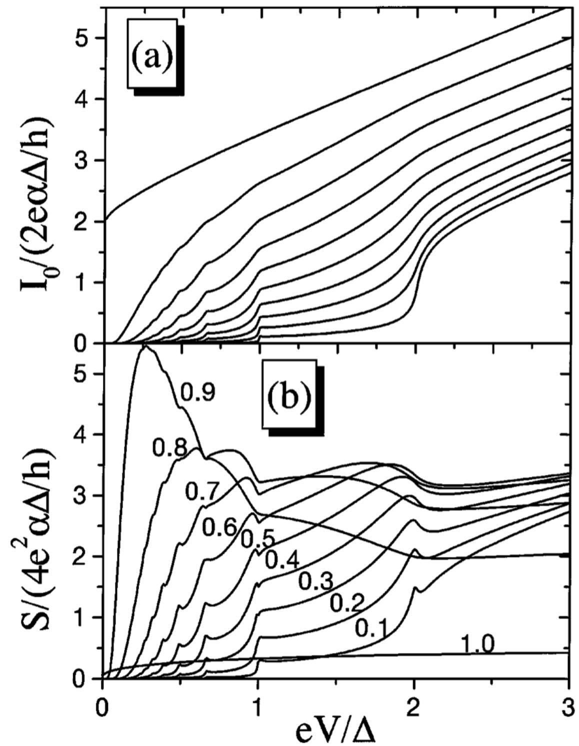
A. Zazunov et al., Phys. Rev. B **94**, 014502 (2016)

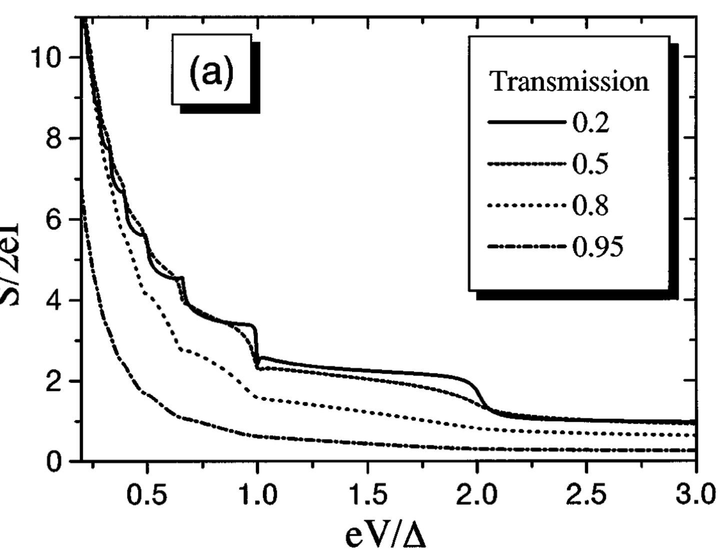
The boundary Green function captures both the continuum properties and the Majorana bound state



2 or more superconductors: BCS case
Multiple Andreev Reflections (MAR)
Cuevas PRL 99

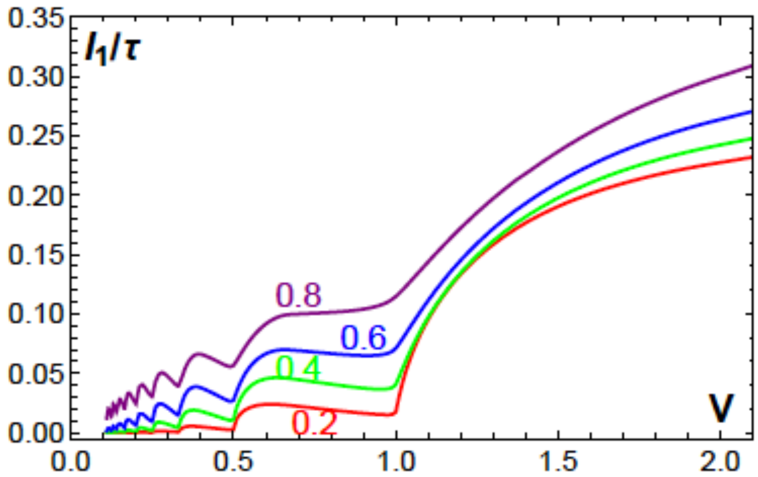




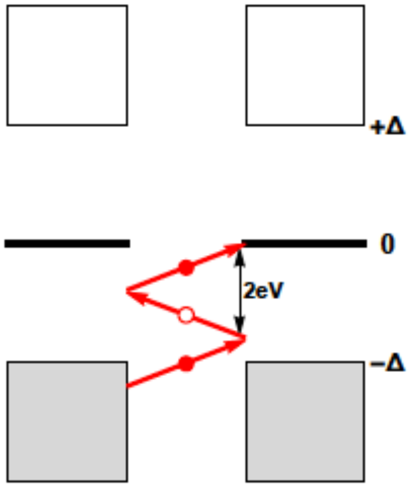


$$S(\omega = 0) \equiv \int_0^{2\pi/\omega_J} \frac{d\bar{t}}{2\pi/\omega_J} \int_{-\infty}^{+\infty} d\tau \; S(\bar{t} + \tau/2, \bar{t} - \tau/2)$$

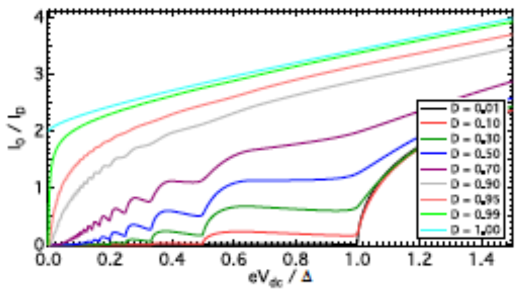
DC current



DC current I_1 in the TS-TS-TS system for different values of transparency τ. Current onsets at $eV = \Delta/n$

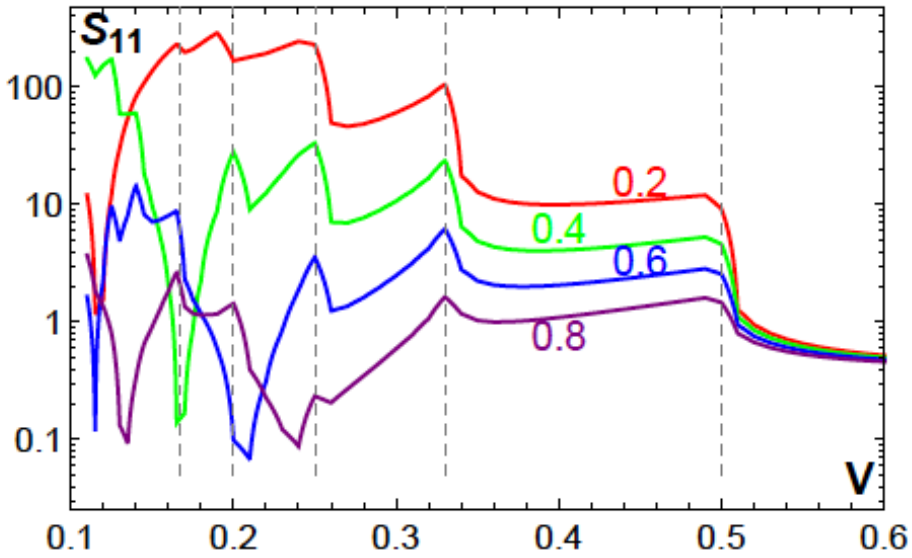


DC current comes from Multiple Andreev Reflection process



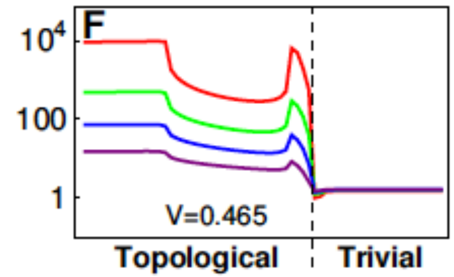
Qualitatively similar to the 2TS case (Badiane et al., C.R. Physique 14, 840 (2013))

Zero-frequency current correlations



Current correlations S_{11} in the TS-TS-TS system for different values of the transparency τ. Note the log scale !

The current correlations are huge (while in the TS-TS case, $S_{11} \sim I_1$). For small V, they increase as transparency is decreased ! Large peaks at the MAR onsets.

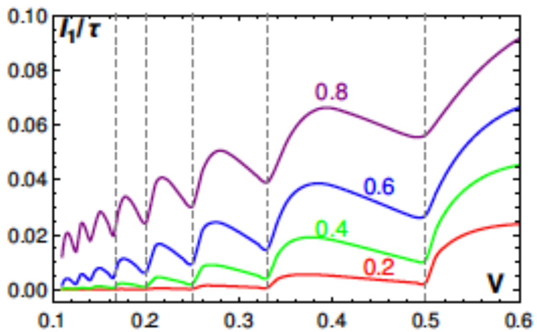


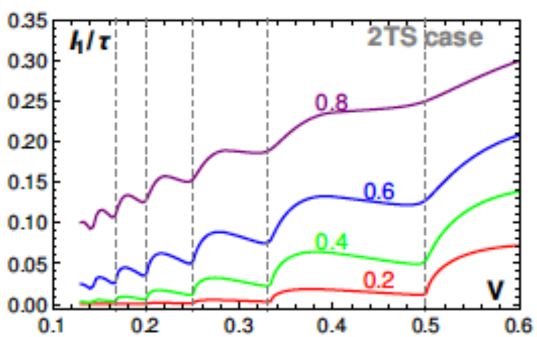
Fano factor as a function of the chemical potential of TS2

The huge values disappear out of the topological regime

3TS vs 2TS

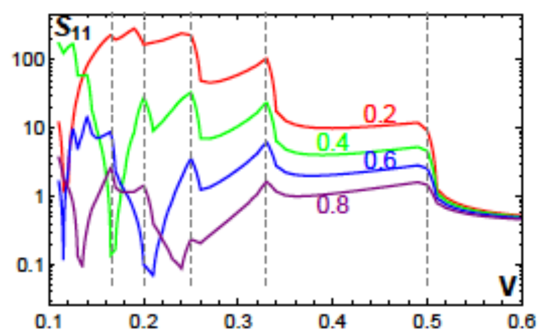
Current

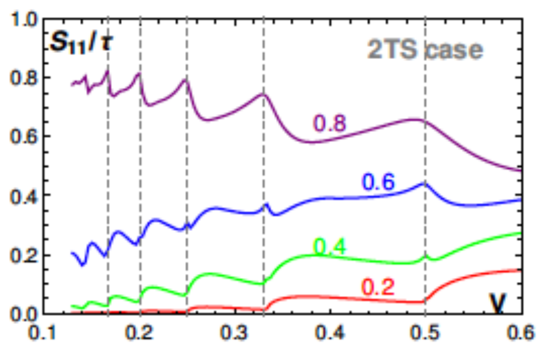




Qualitatively similar results for the current

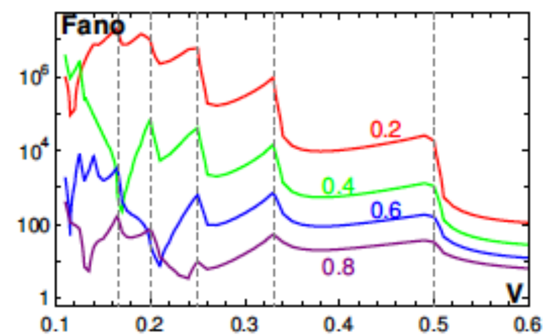
Noise

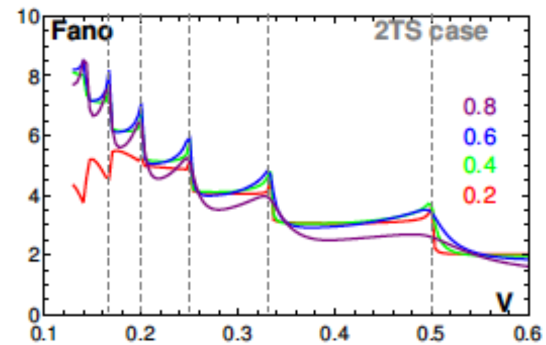




Widly different scales for the noise

Fano factor





Fano factor $\sim 1/V$ for 2TS, orders of magnitude larger for 3TS

Atomic limit approximation - 1

Large Δ approximation

For small V and small transparency τ, Δ is the largest energy scale of the problem $\Rightarrow$ neglect the continuum of quasiparticles states above the gap. Only the majorana bound states γ_0, γ_1, γ_2 are left.

Hamiltonian 1

Hamiltonian becomes :
$H = i\sqrt{2}\Omega(t)(\gamma_1 - \gamma_2)\gamma_0$ with
$\Omega(t) = \lambda\Delta\sin(Vt)$

Hamiltonian 2

Define :
$\gamma_\pm = 1/\sqrt{2}(\gamma_1 \pm \gamma_2)$
$d = 1/\sqrt{2}(\gamma_- + i\gamma_0)$ then

$$H = \Omega(t)(2d^\dagger d - 1)$$
$$I = \lambda_1\Delta\cos(Vt)[1/\sqrt{2}(2d^\dagger d - 1) + \gamma_+(d - d^\dagger)]$$

Simple Hamiltonian with a single fermion - parity $(-1)^{d^\dagger d}$ is conserved

Current operator has non-trivial coupling to the MBS γ_+

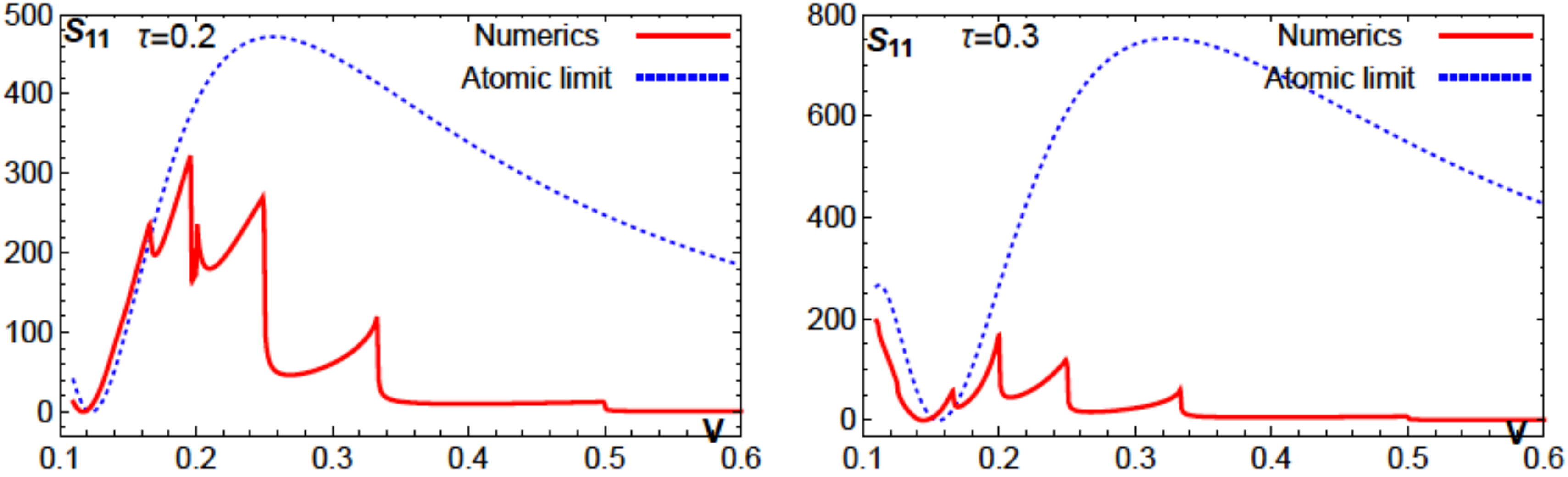
Atomic limit approximation - 2

S_{11} formula

Formula for the current correlations :

$$S_{11}^{(\mathrm{at})} = \frac{\lambda^2\Delta^2}{4\eta} J_1^2\left(2\lambda\Delta/V\right)$$

where $\eta \ll 1$ is the intrinsic parity relaxation rate



The atomic limit formula captures the very small V limit, and gives the overall scale. But misses all the complex structures in the noise.

Beyond atomic limit : effect of MAR current

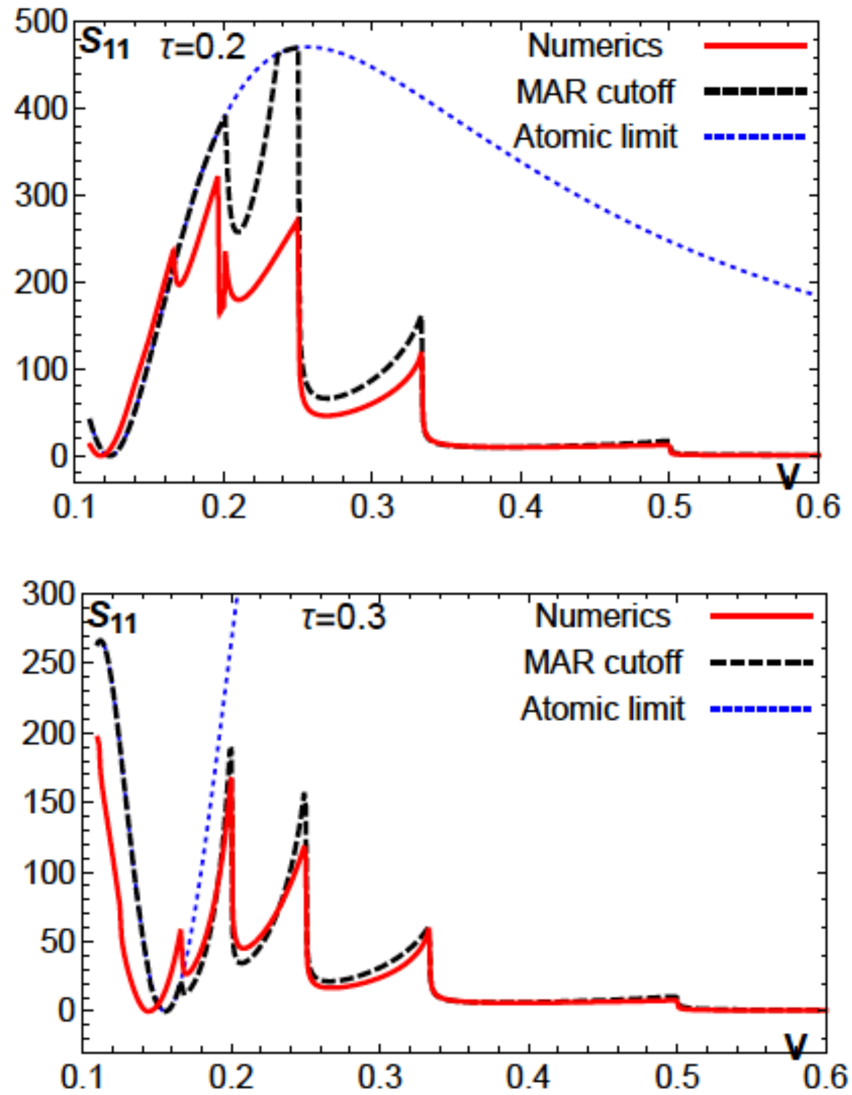
Continuum quasiparticles

Taking into the continuum particle states, we see that MAR processes comes from coupling between d fermion and continuum states
$\Rightarrow$ MAR processes change the parity of the d fermion

MAR coherence time

MAR processes will lead to loss of coherence for the fermion dynamics
$\Rightarrow$ cutoff time for the fluctuations.
$T_{MAR}(V) = (1 + \Delta/V)/I(V)$

Effective parity relaxation rate :
$\eta \to \eta_{eff}(V) = \max(T_{MAR}^{-1}(V), \eta)$



Conclusion

- Giant shot noise S_{11} in a out-of-equilibrium TS-TS-TS setup
- Large fluctuations due to the presence of a zero-mode
  Coherence limited by MAR current
- Multi-terminal systems allow to probe unique properties which are not accessible in a 2-terminal setup

Reference

• T. Jonckheere, J. Rech, A. Zazunov, R. Egger, A. Levy Yeyati, T. Martin
Phys. Rev. Lett. **122**, 097003 (2019)

GENERAL CONCLUSIONS:

- Multi-terminal nanodevices are interesting !
- Keldysh Green's function approach to hybrid N-TS systems, treat: below/above gap transport (generalizations possible).
- HBT TS beam splitter leads to noise crossed correlations which are either negative or positive (+/- sign of voltages).
- Finite frequency noise of an NTS junction has a distincive plateau.
- 3 terminal TS junction bear giant noise associated to the presence of a zero mode.

Tadanori Yokoo
photographer of Y junctions

Finite size TS Green's function

$$g^{R/A}(\omega) = \omega \tanh(\zeta_\omega L) \frac{\zeta_\omega \sigma_0 - \tanh(\zeta_\omega L) \Delta \sigma_x}{(\omega \pm i0^+)^2 - \epsilon_\omega^2}$$