Spin liquids in insulators and metals

ICTP Summer School
Advances in Condensed Matter Physics:
New trends and Materials in Quantum Technologies
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Talk online: sachdev.physics.harvard.edu

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- Combination of band topology and emergent gauge fields leads to exotic new possibilities (non-Abelian bulk anyons)

1. Resonating valence bonds

The Z_2 spin liquid

2. SU(2) gauge theory of fluctuating antiferromagnetism on the triangular lattice

The Z_2 spin liquid

3. Electron-doped cuprates

Higgs phase with topological order: Fermi surface reconstruction without translational symmetry breaking

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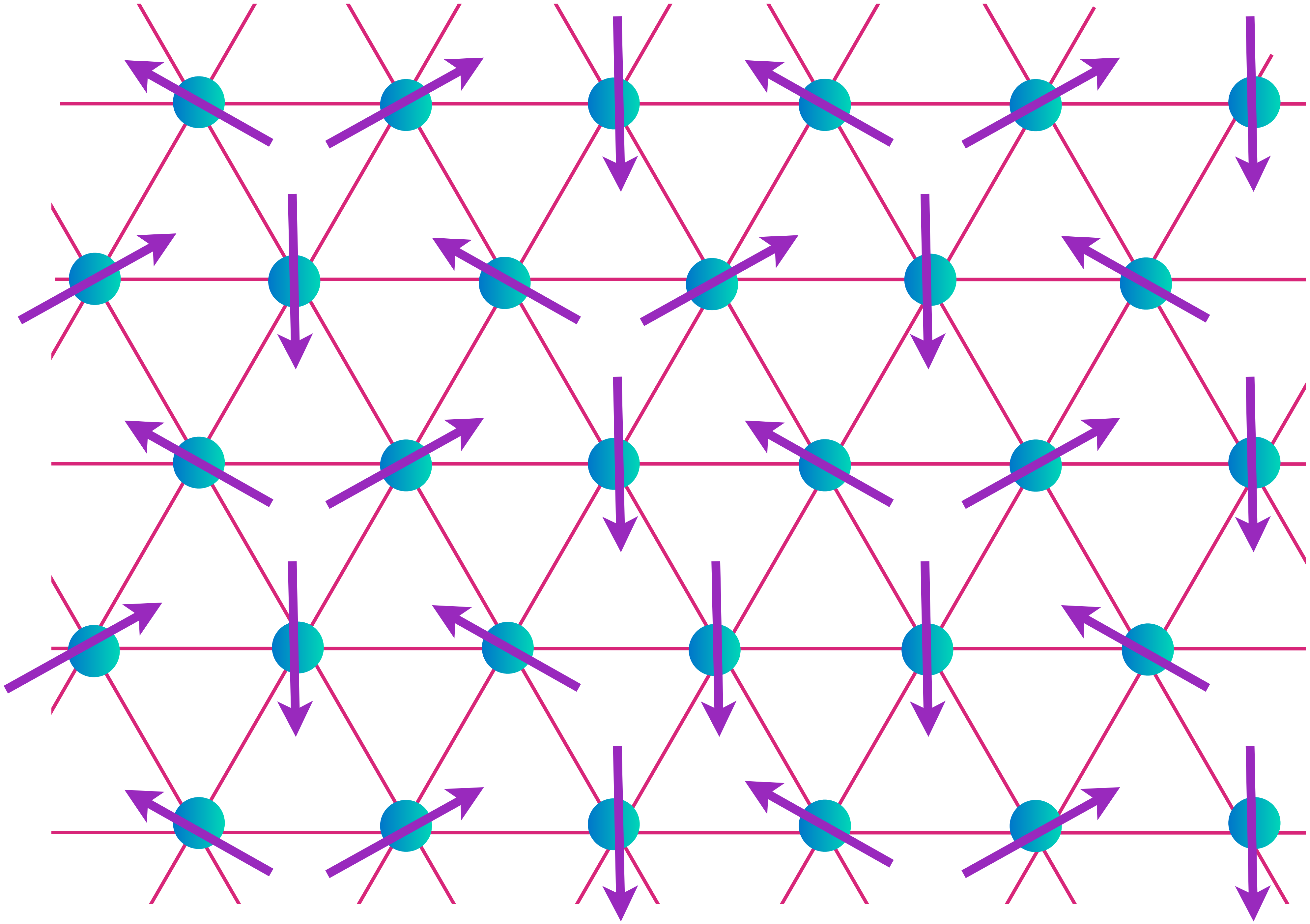
Mott insulator: Triangular lattice antiferromagnet

$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$



Nearest-neighbor model has non-collinear Neel order

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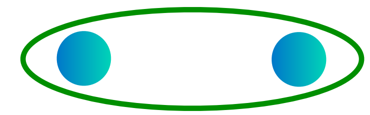
Spin liquid obtained in a generalized spin model with S=1/2 per unit cell



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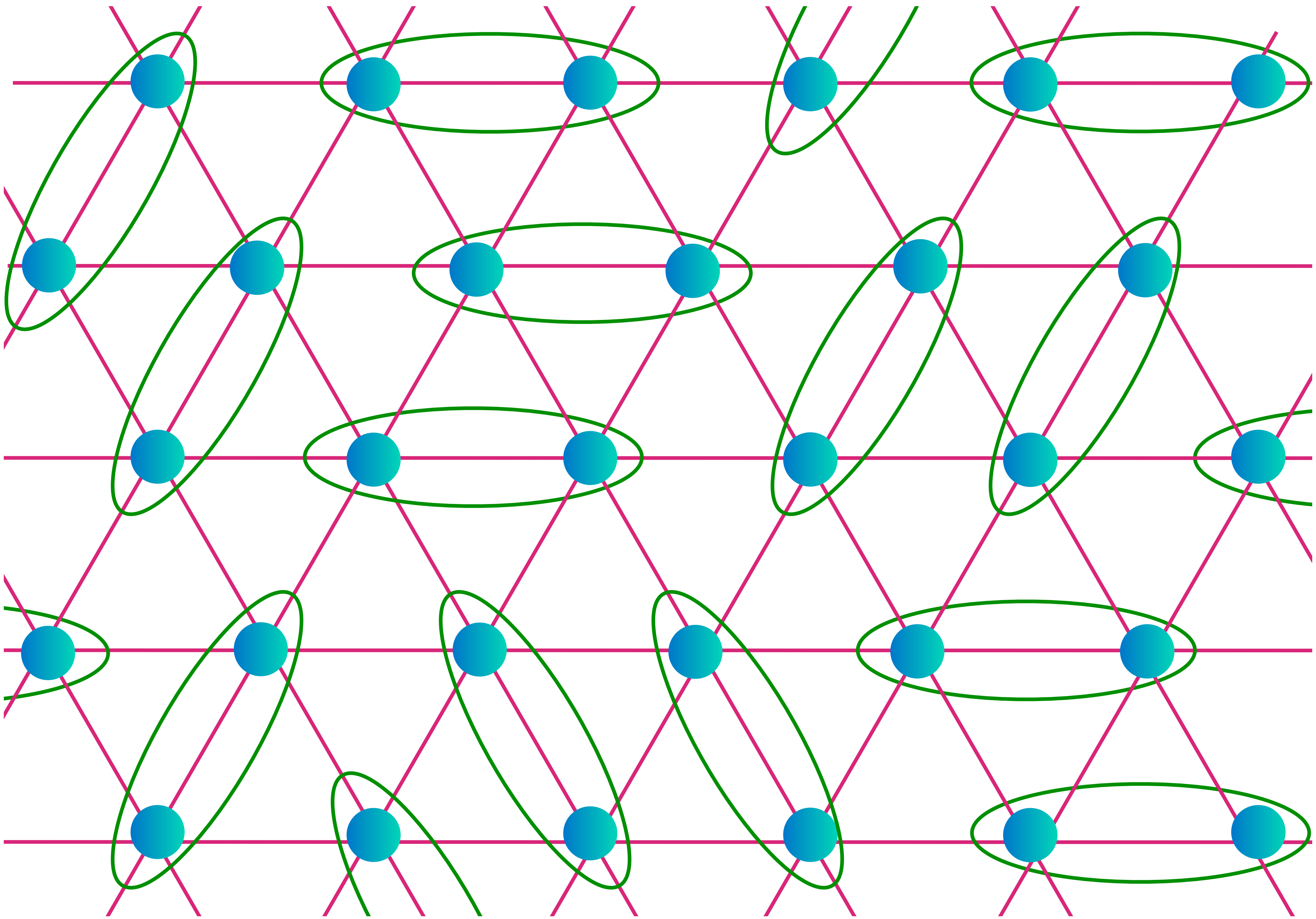

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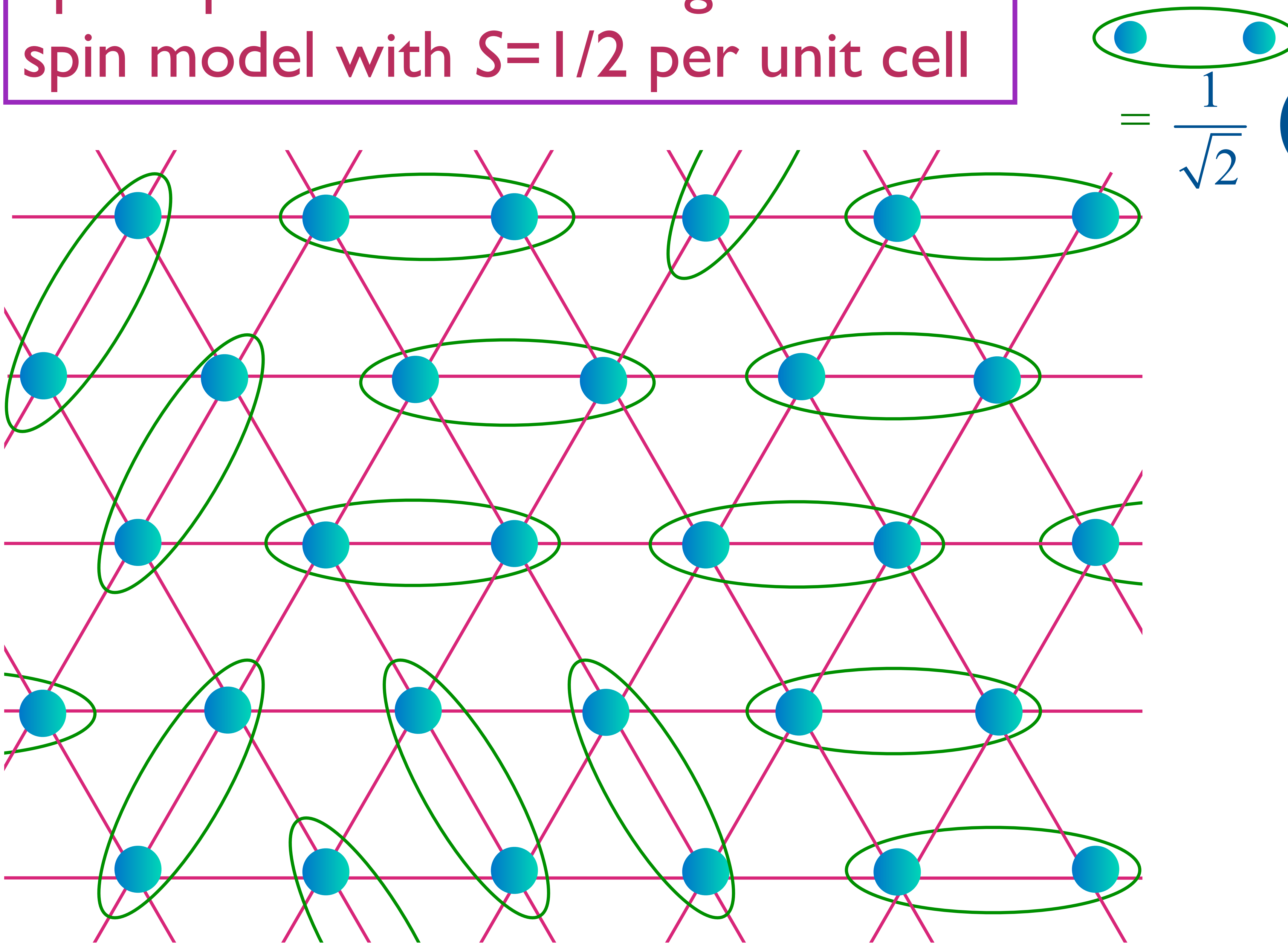

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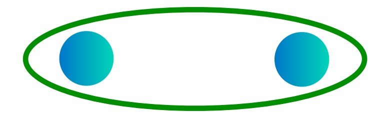

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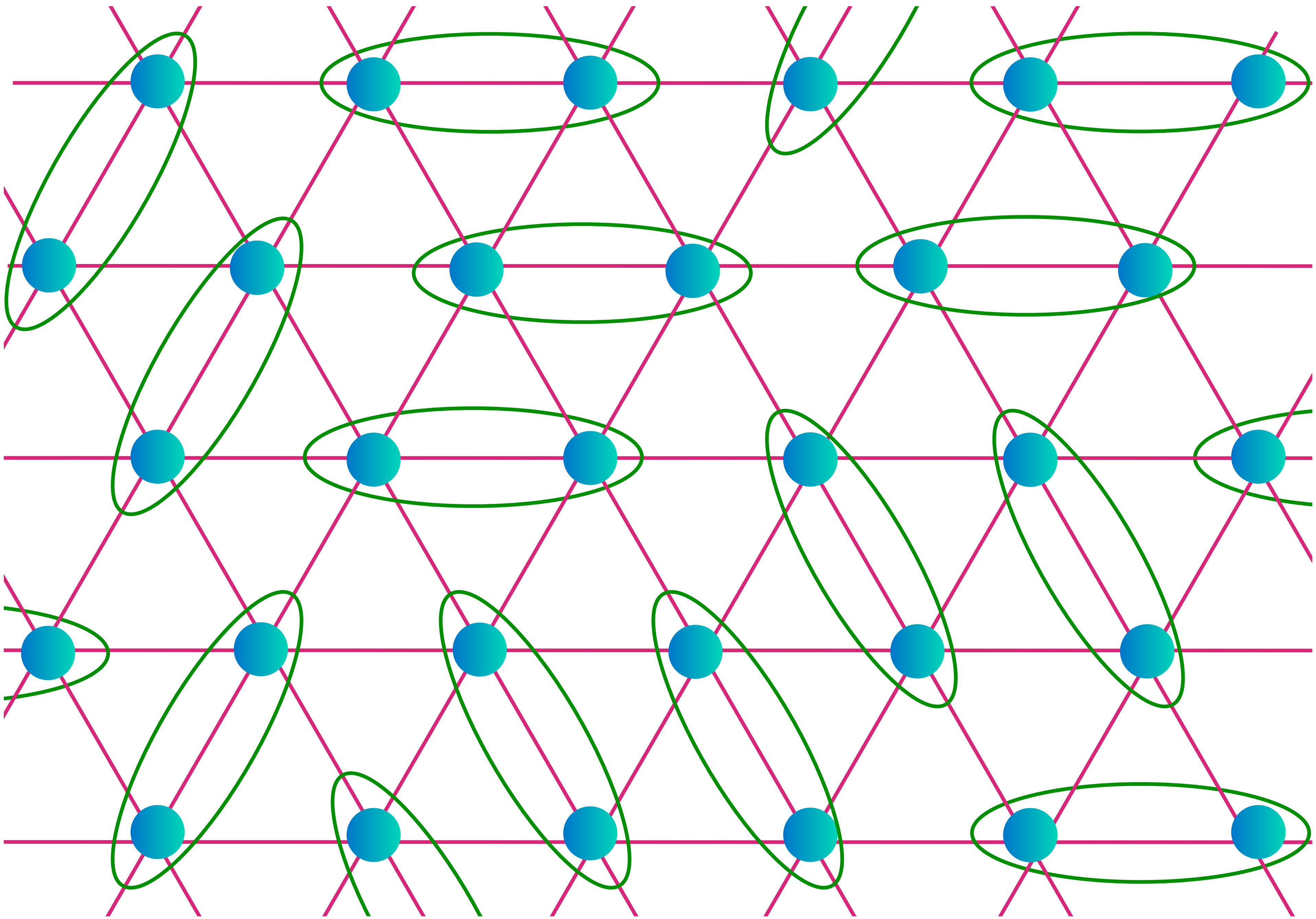

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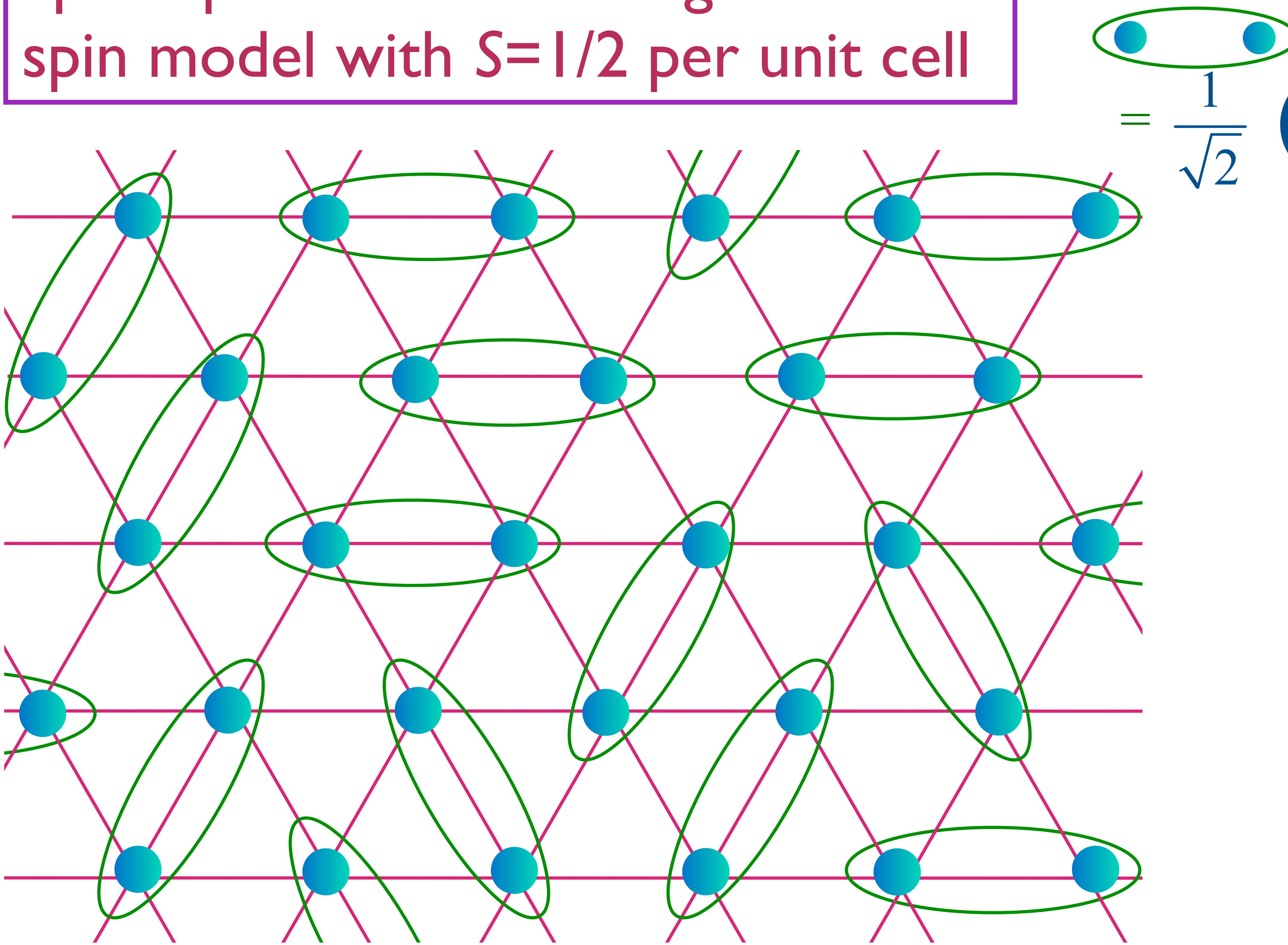

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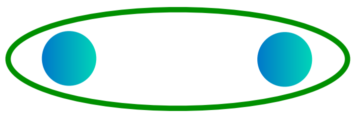

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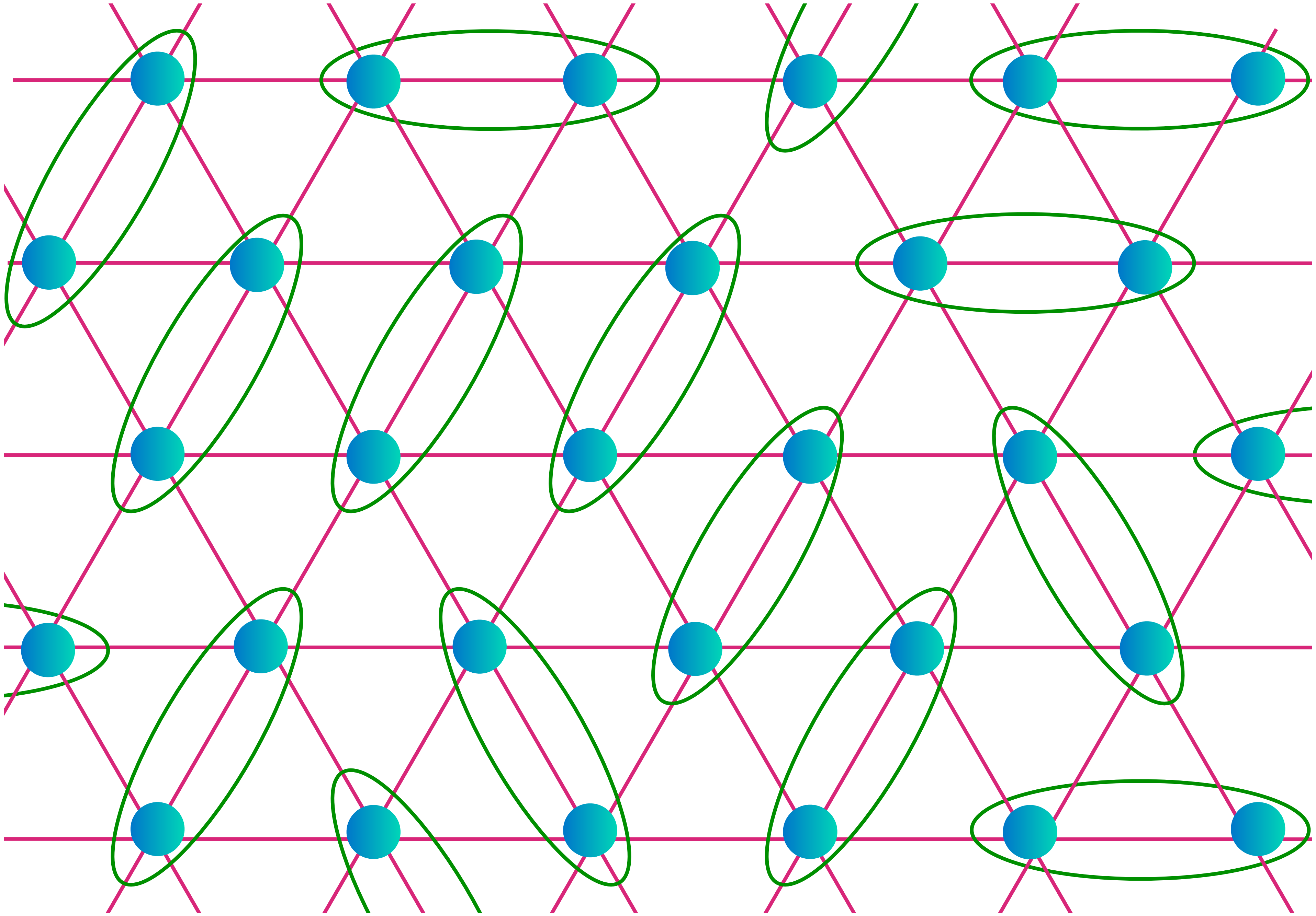

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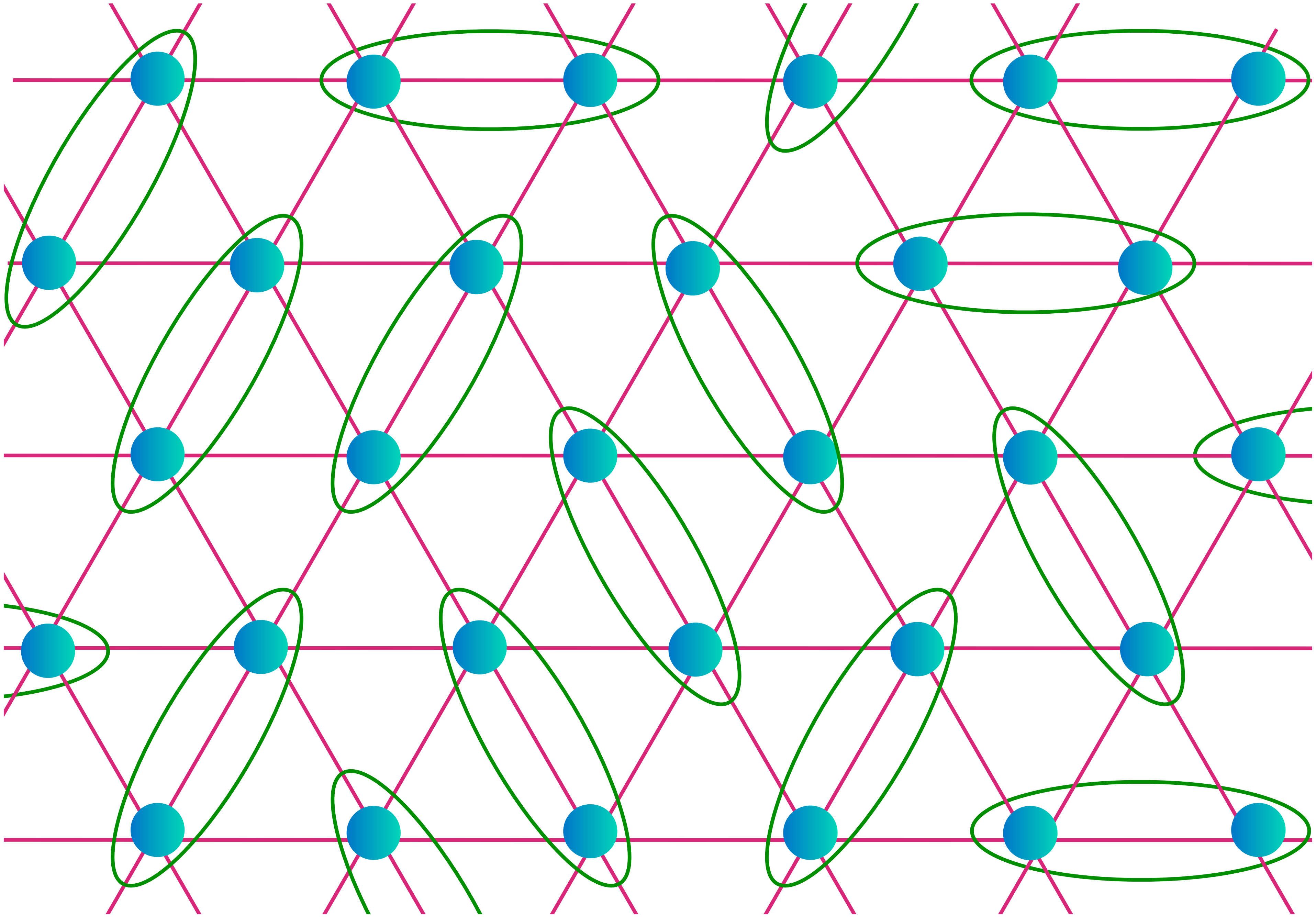

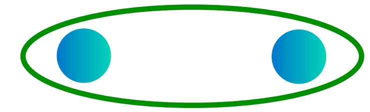

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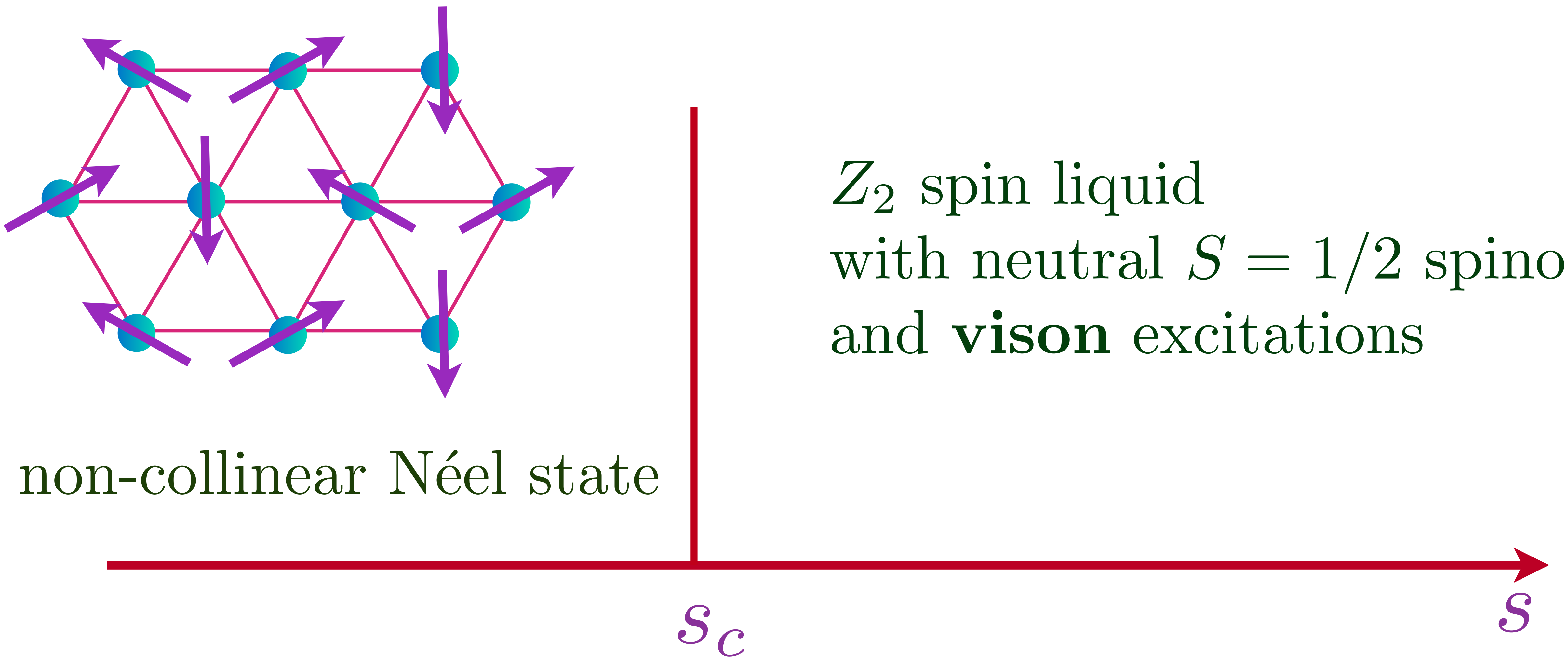

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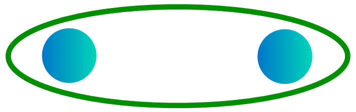
Excitations of the Z_2 Spin liquid



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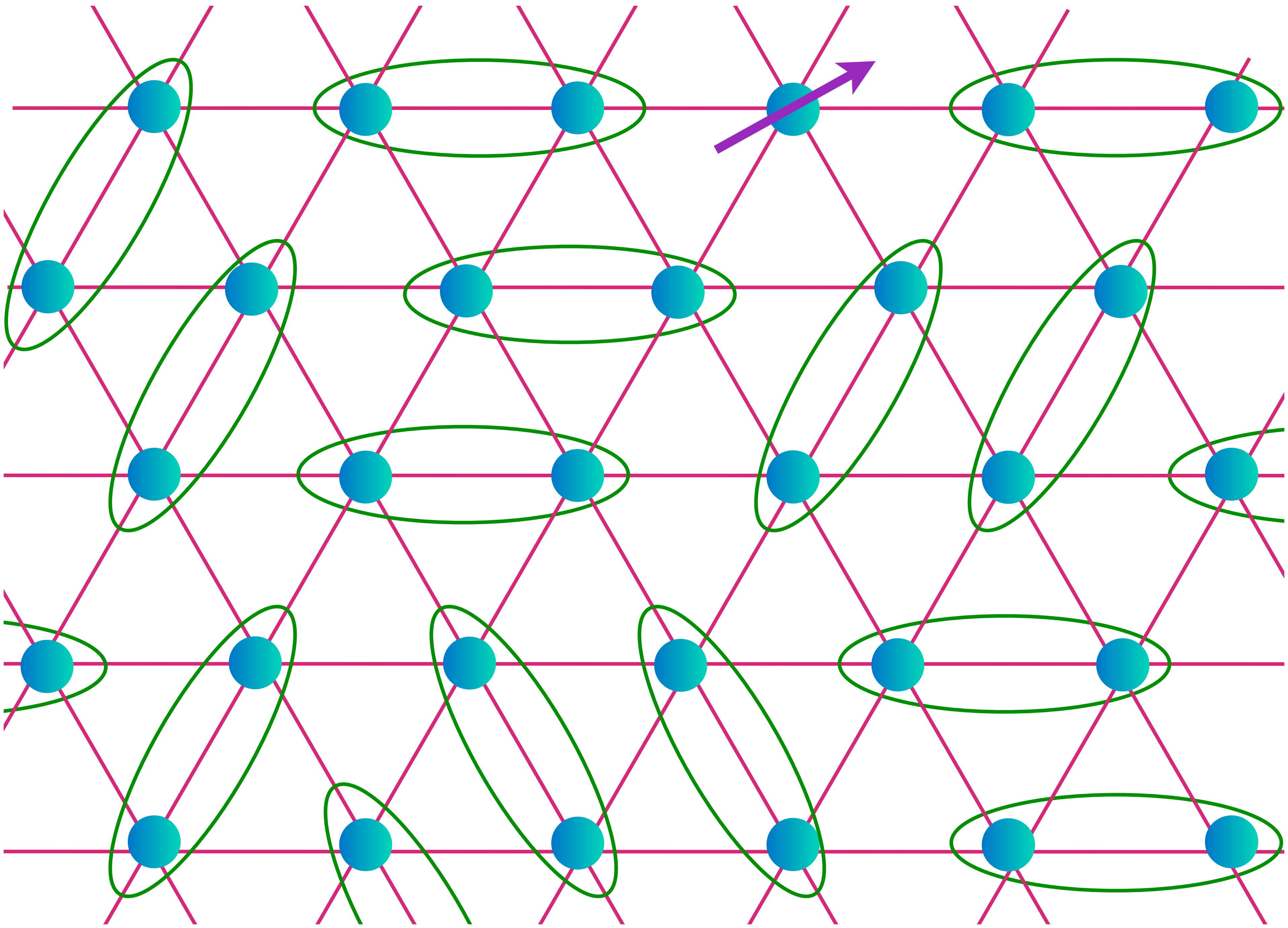
Spinon: S=1/2, charge 0

e (boson) or ϵ (fermion) particle



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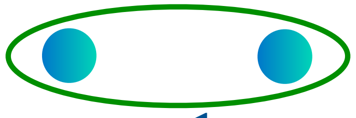

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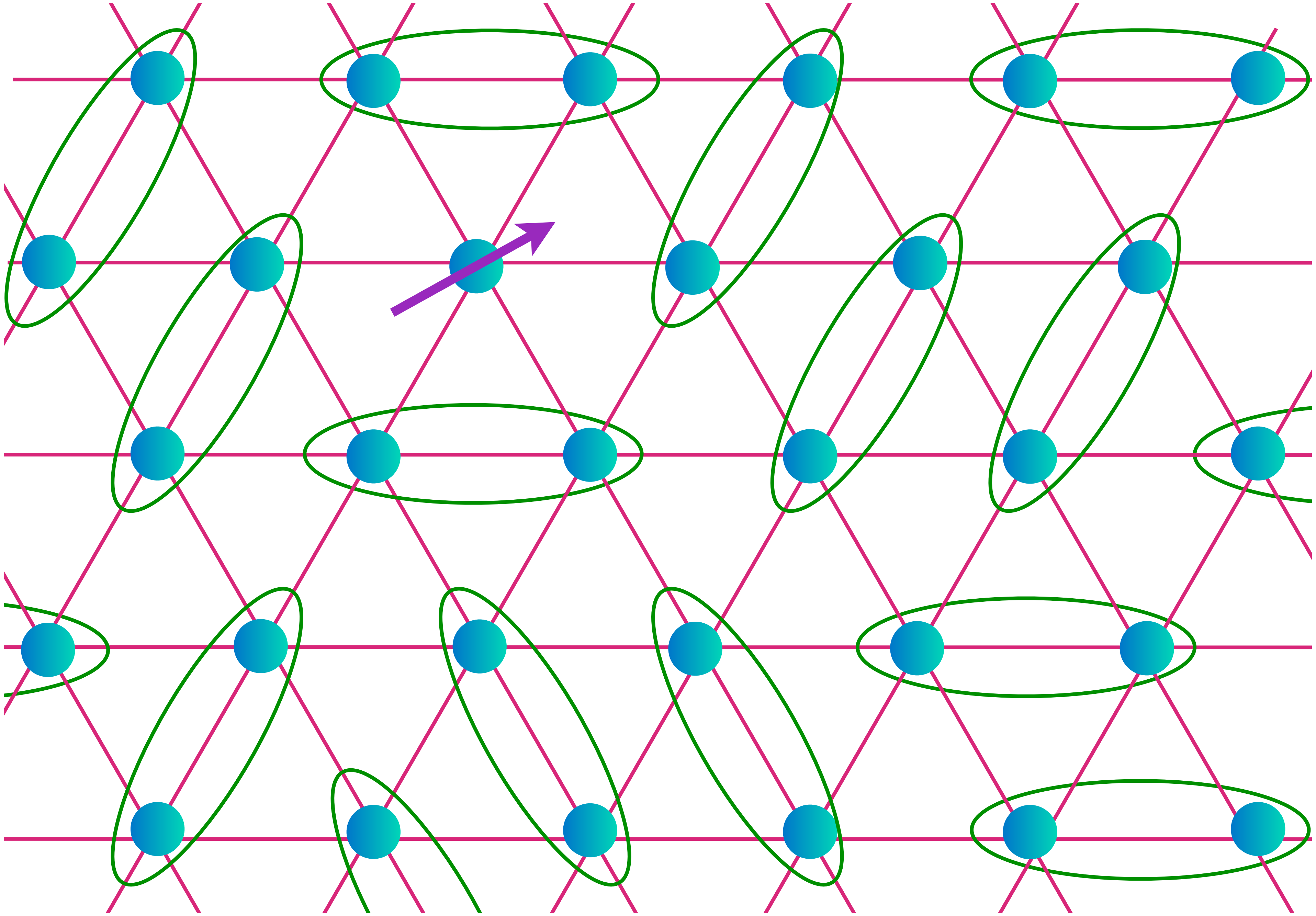

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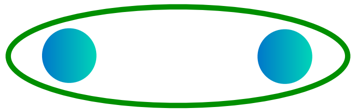

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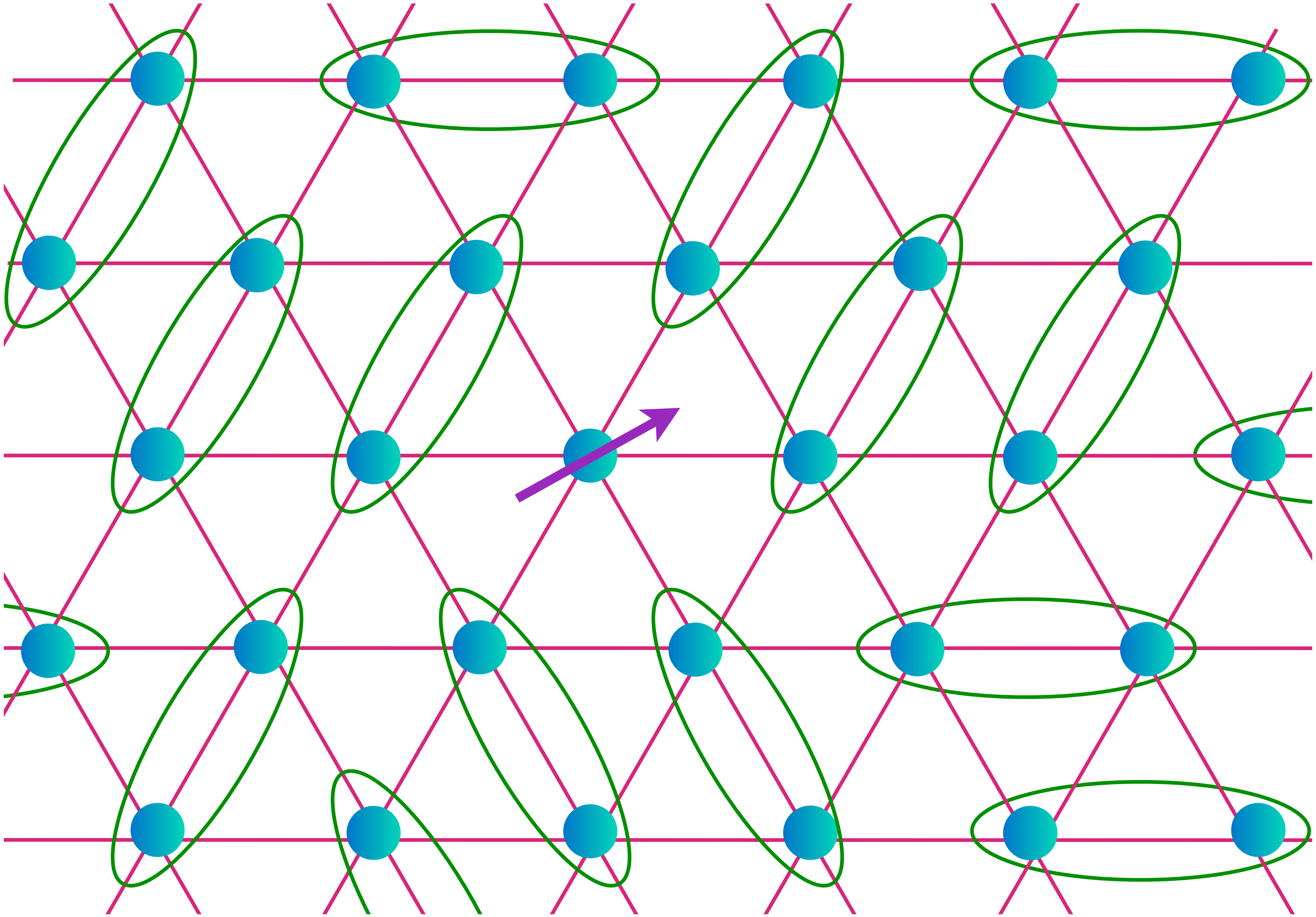

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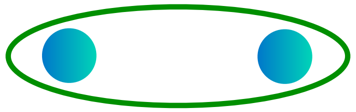

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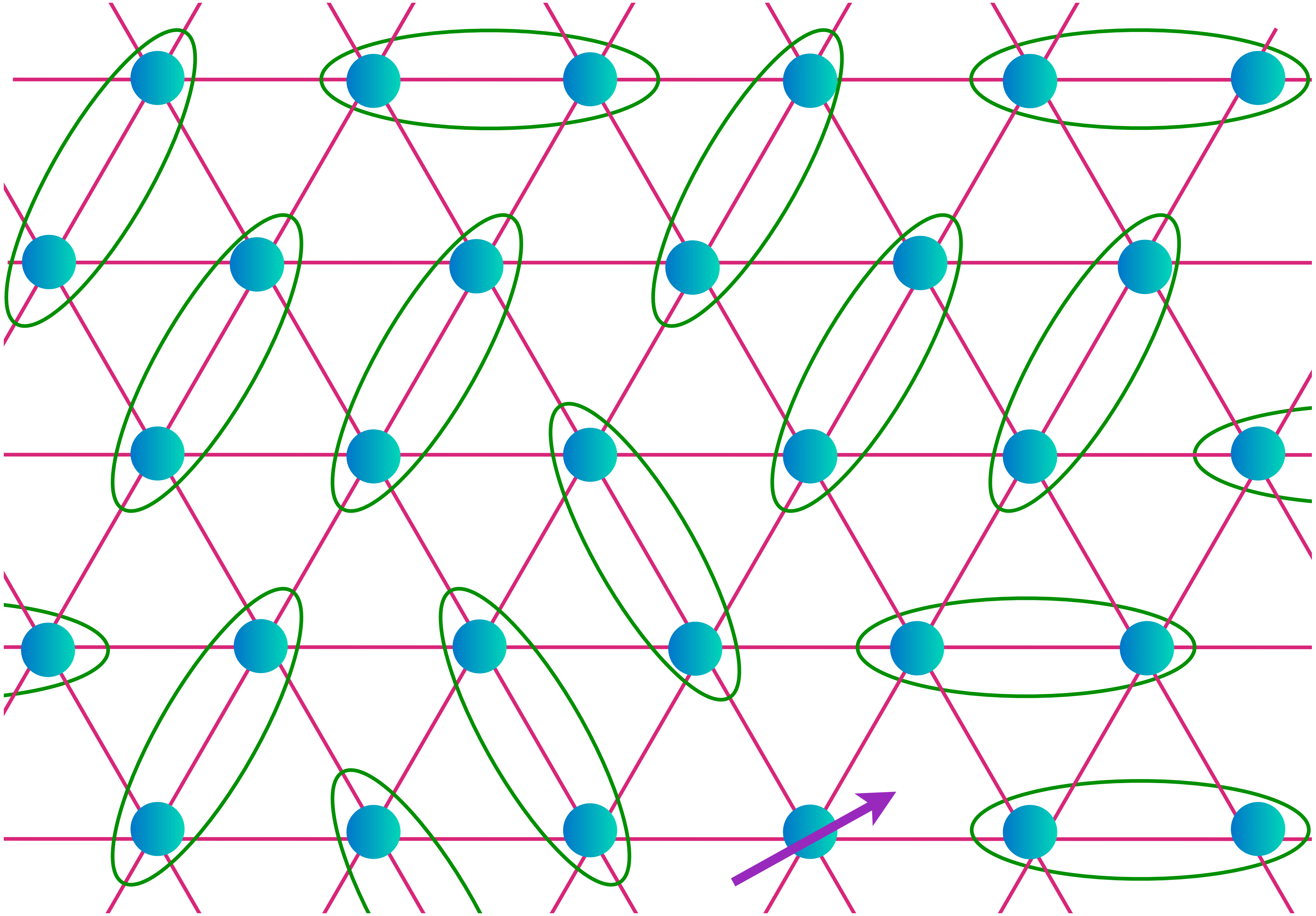

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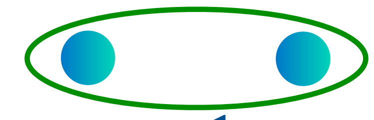
A vison

m (boson) particle



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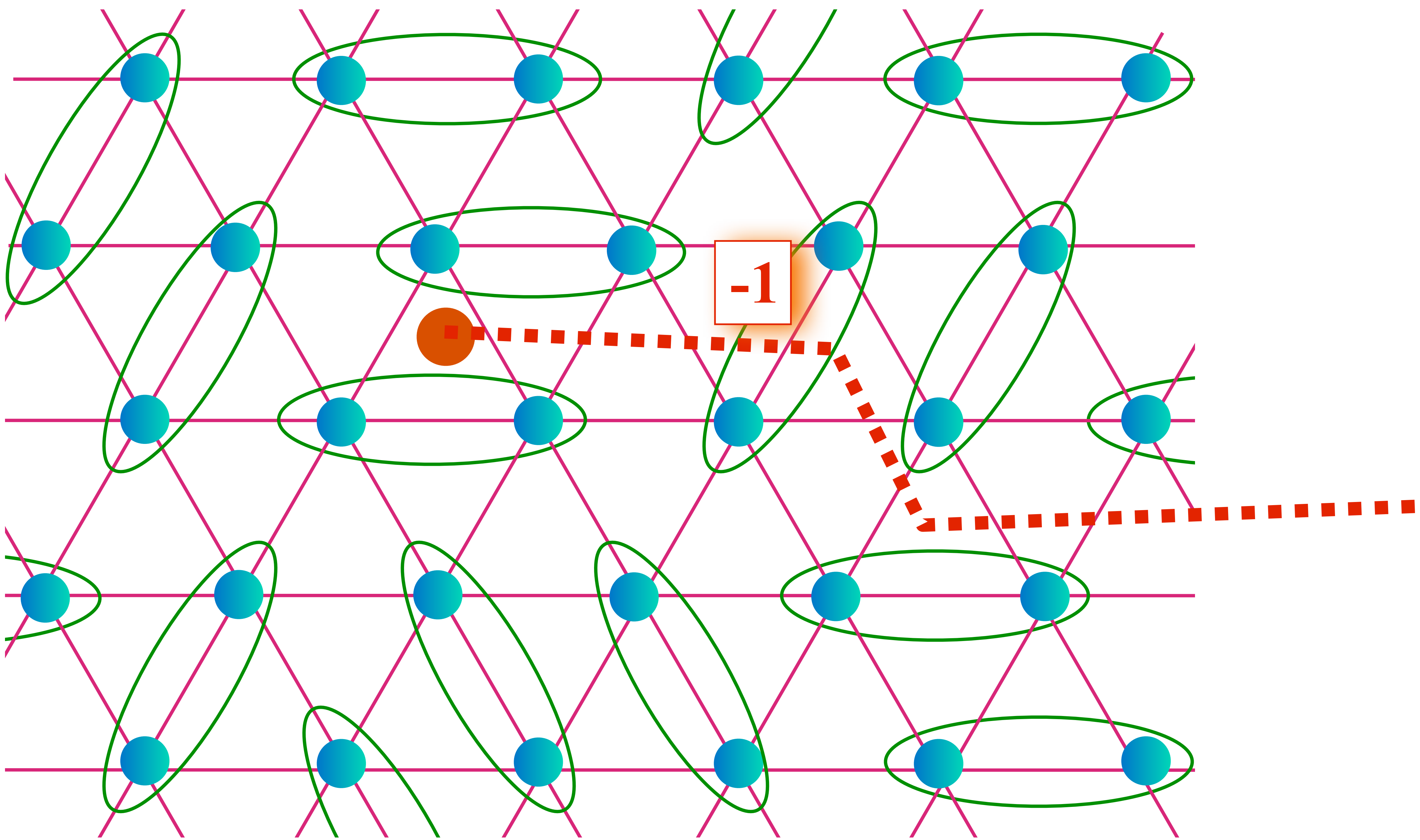

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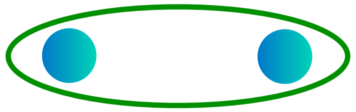

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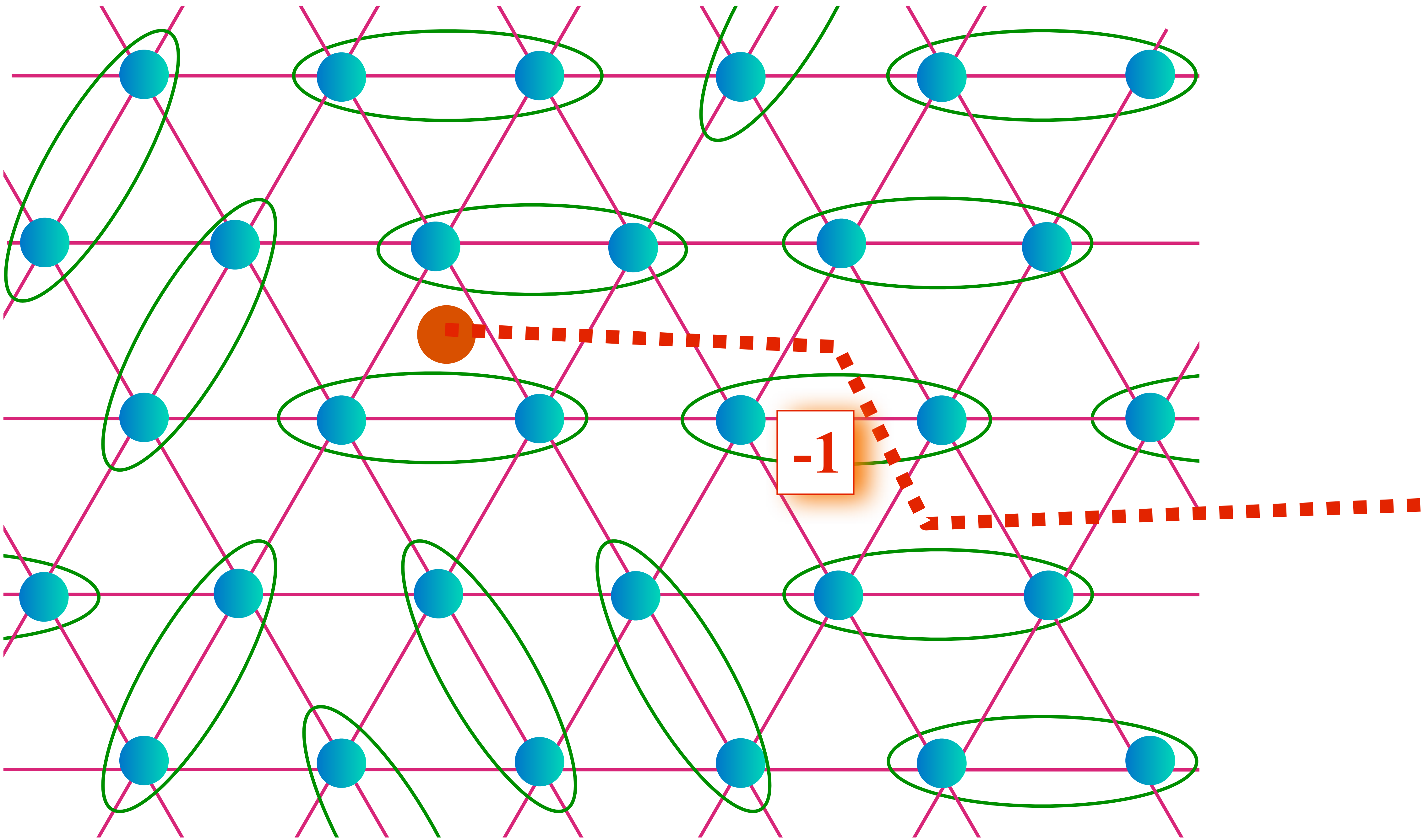

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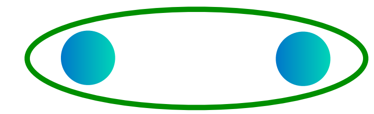

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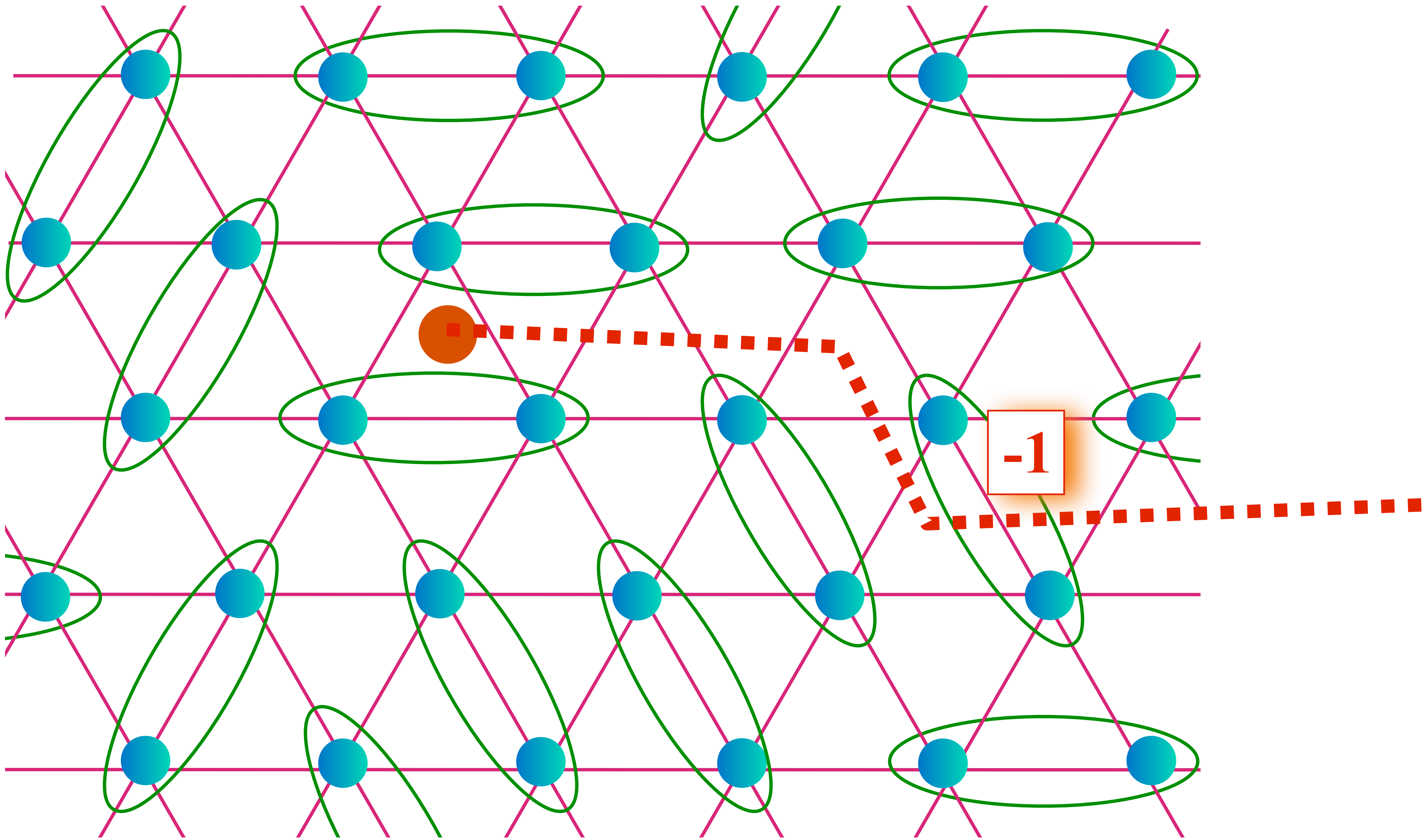

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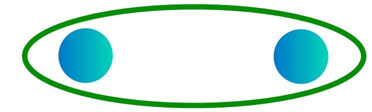

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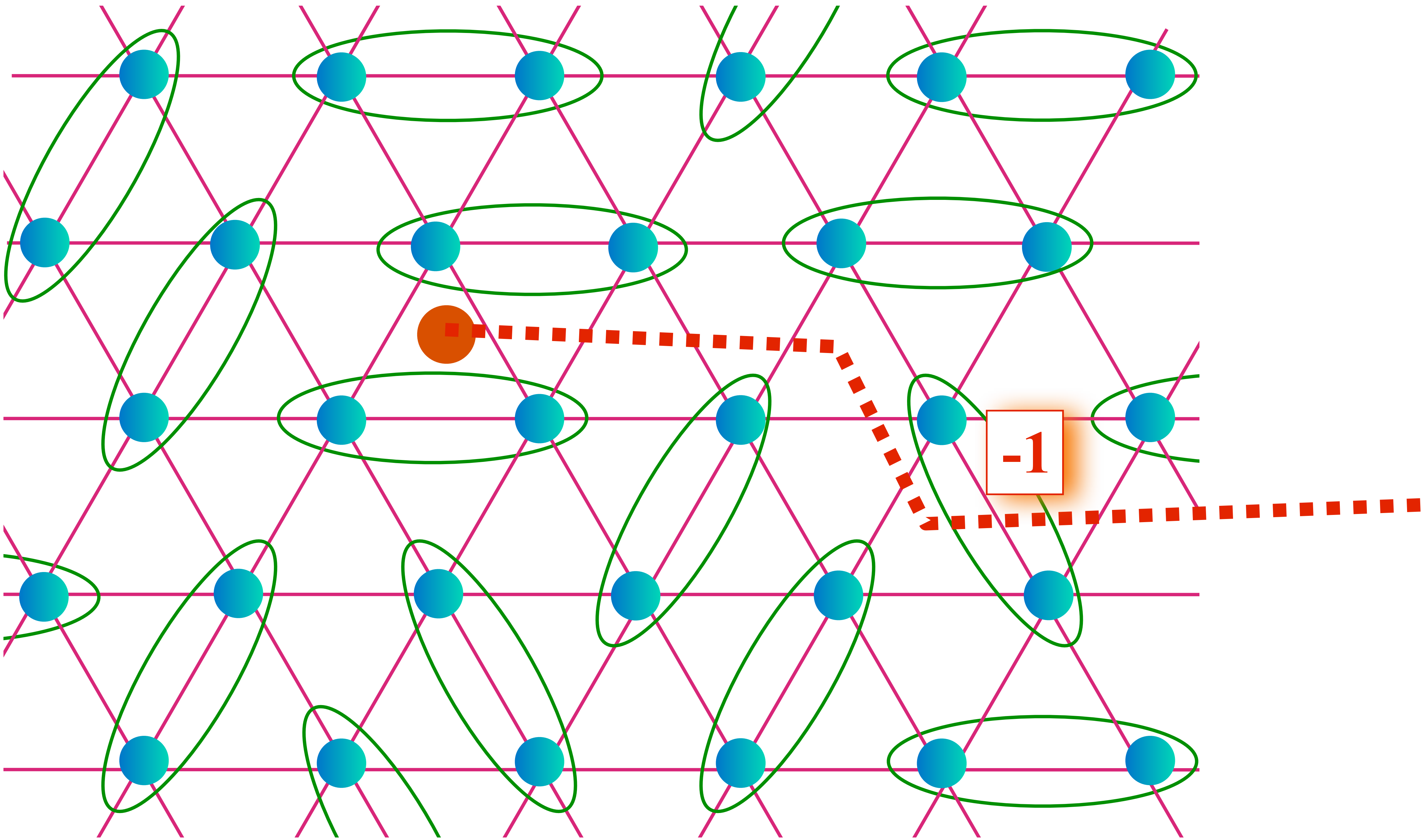

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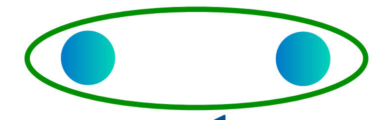

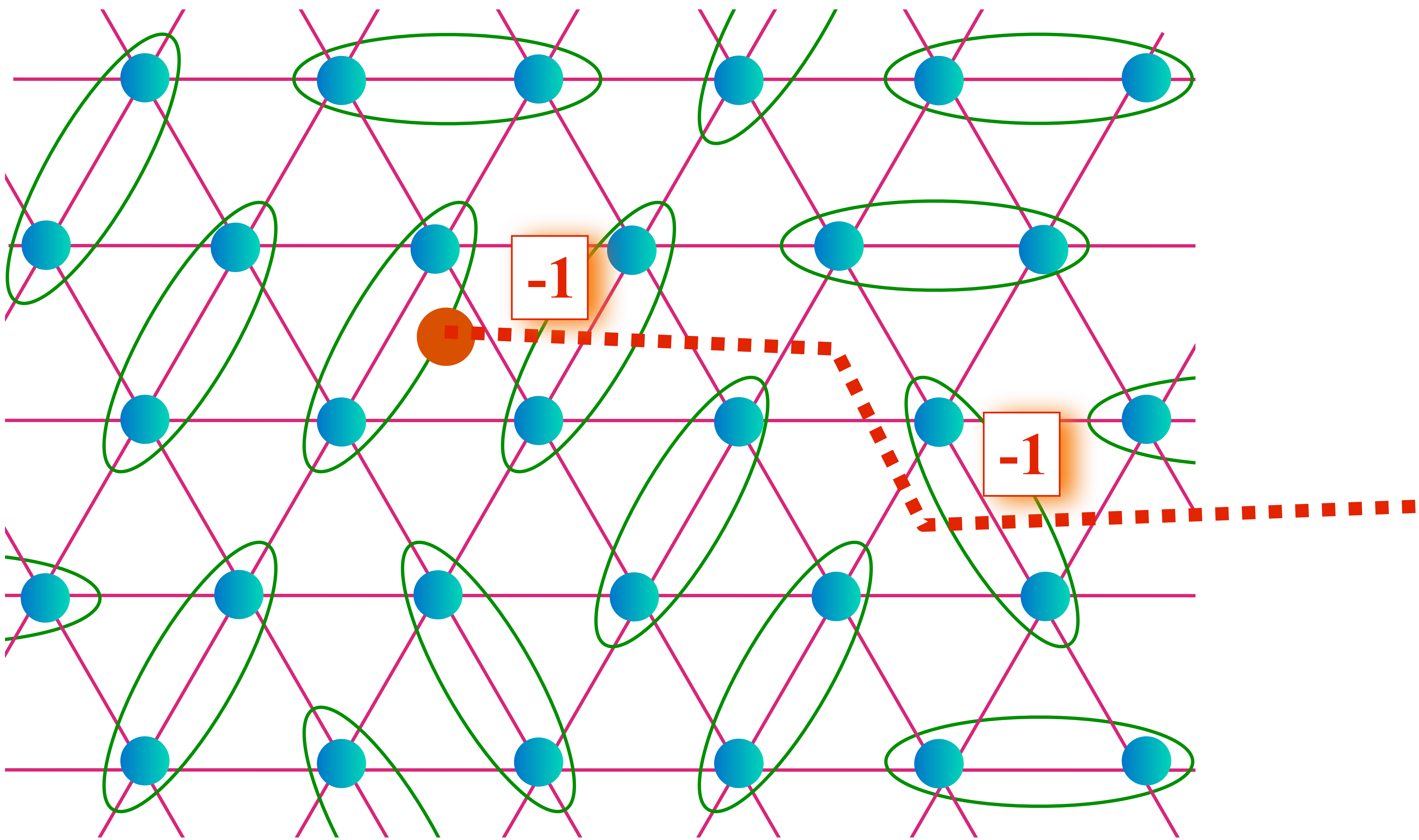

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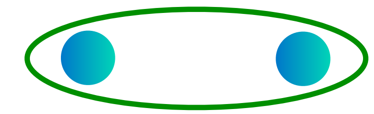

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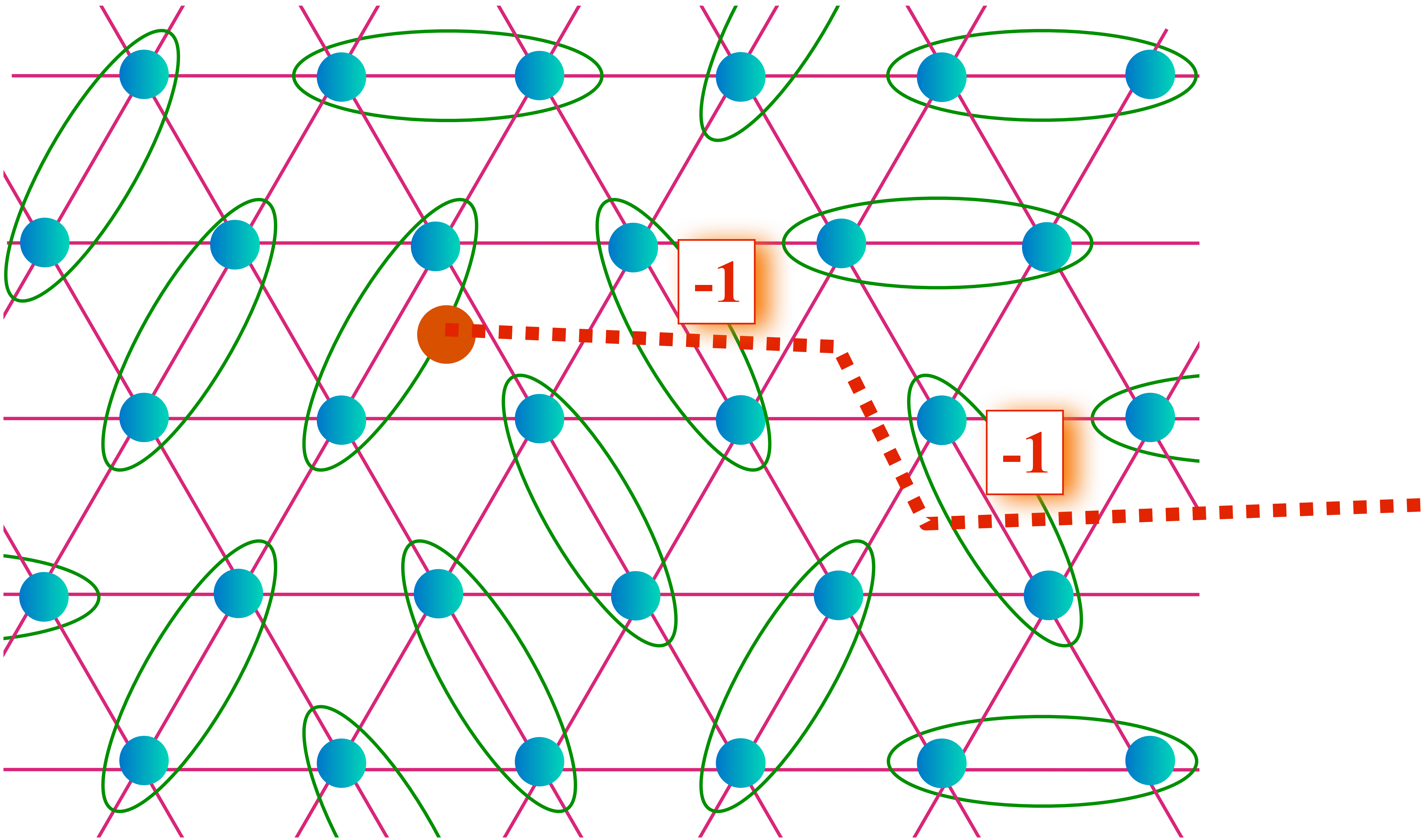

Topological order in the Z_2 spin liquid ground state

4-fold degeneracy on the torus

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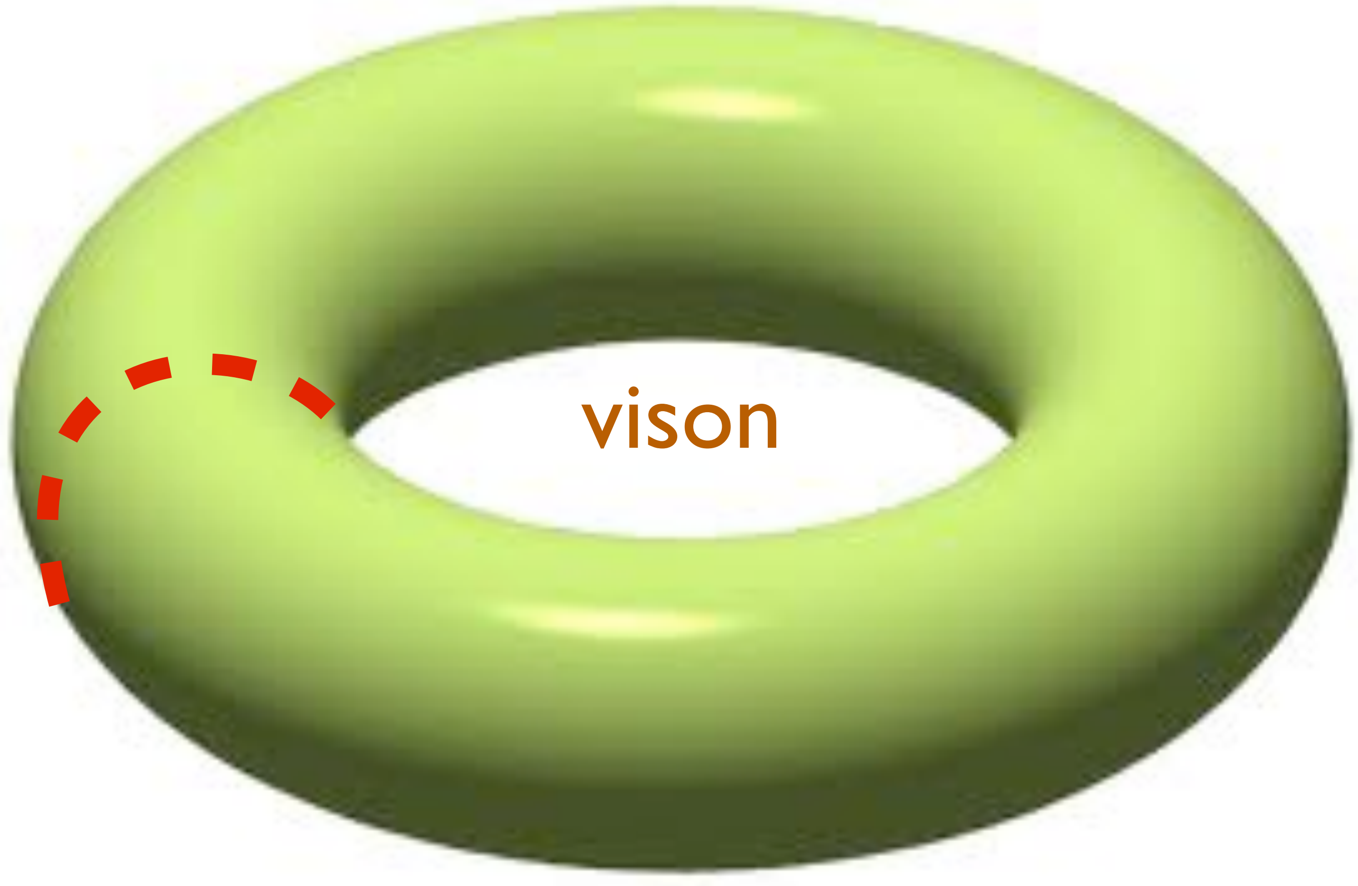

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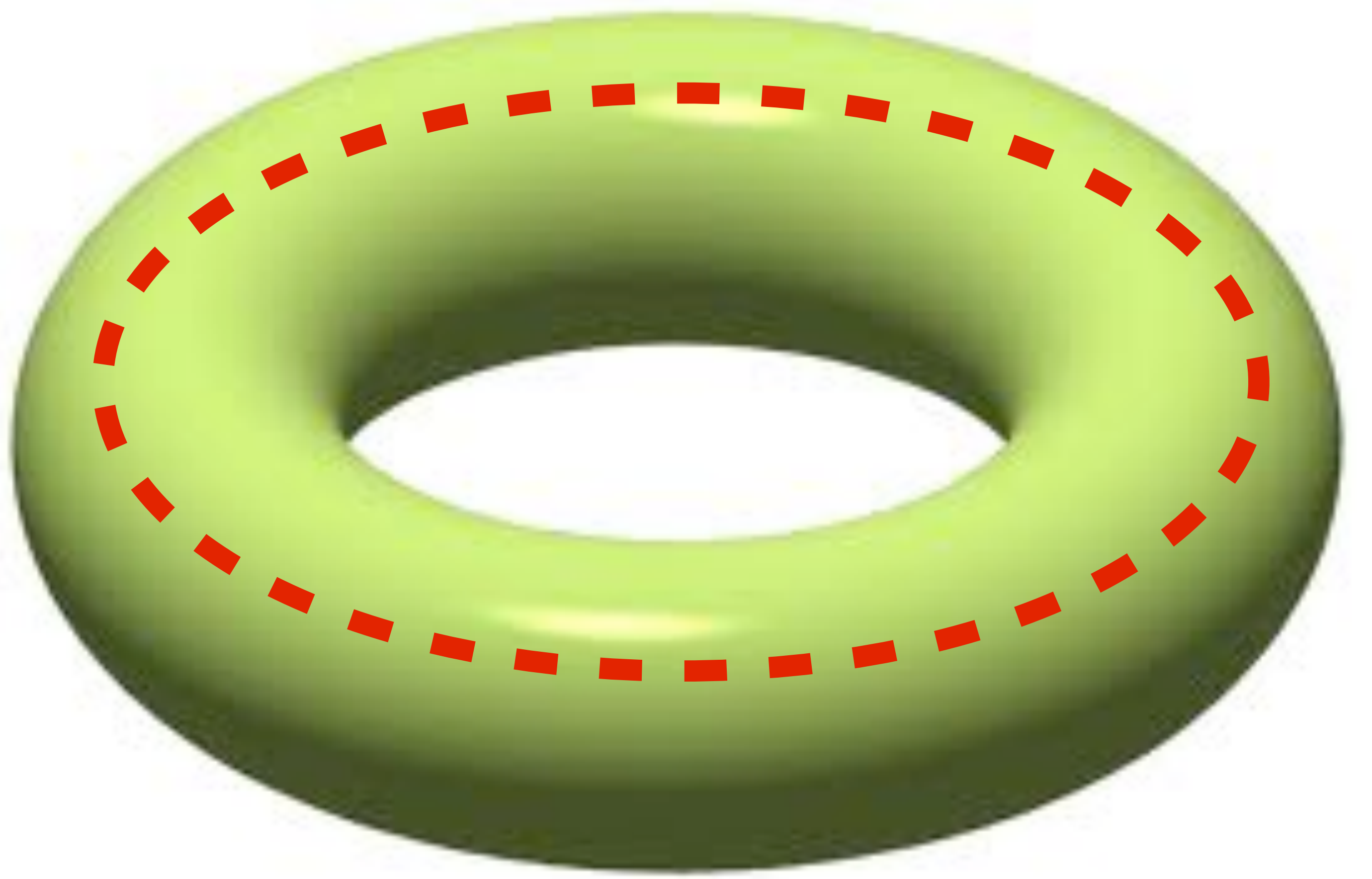

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- Protected edge states do not exist in general, but could appear in the presence of symmetries.

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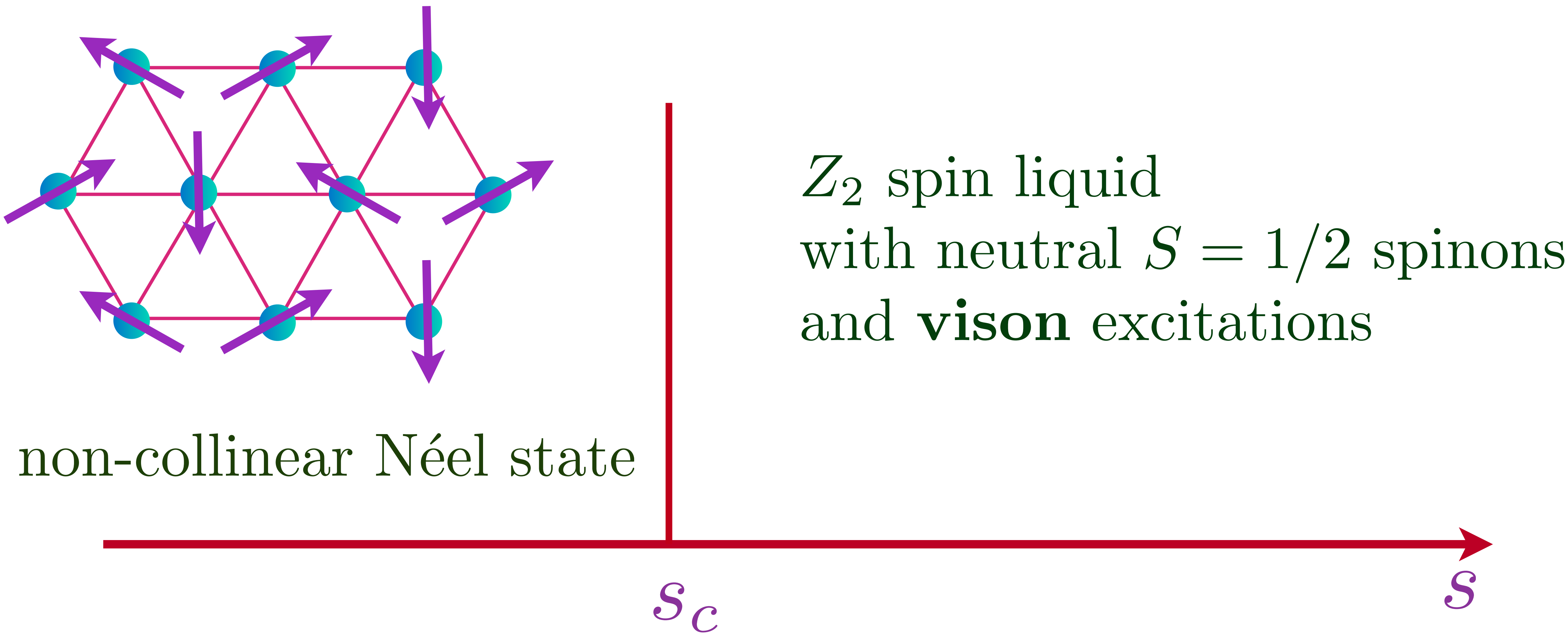

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The Hubbard Model

$$H = -\sum_{i<j} t_{ij} c^\dagger_{i\alpha} c_{j\alpha} + U \sum_i \left(n_{i\uparrow} - \frac{1}{2} \right) \left(n_{i\downarrow} - \frac{1}{2} \right) - \mu \sum_i c^\dagger_{i\alpha} c_{i\alpha}$$

$t_{ij} \to$ "hopping". $U \to$ local repulsion, $\mu \to$ chemical potential

Spin index $\alpha = \uparrow, \downarrow$

$$n_{i\alpha} = c^\dagger_{i\alpha} c_{i\alpha}$$

$$c^\dagger_{i\alpha} c_{j\beta} + c_{j\beta} c^\dagger_{i\alpha} = \delta_{ij} \delta_{\alpha\beta}$$
$$c_{i\alpha} c_{j\beta} + c_{j\beta} c_{i\alpha} = 0$$

First study on the triangular lattice

We use the operator equation (valid on each site i):

$$U\left(n_{\uparrow}-\frac{1}{2}\right)\left(n_{\downarrow}-\frac{1}{2}\right)=-\frac{2U}{3}\vec{S}^{2}+\frac{U}{4}$$

Then we decouple the interaction via

$$\exp\left(\frac{2U}{3}\sum_{i}\int d\tau\vec{S}_{i}^{2}\right)=\int\mathcal{D}\vec{\Phi}_{i}(\tau)\exp\left(-\sum_{i}\int d\tau\left[\frac{3}{8U}\vec{\Phi}_{i}^{2}-\vec{\Phi}_{i}\vec{S}_{i}\right]\right)$$

In this manner, we obtain the "spin-fermion" model

$$\mathcal{Z} = \int \mathcal{D}c_\alpha \mathcal{D}\vec{\Phi} \exp\left(-\mathcal{S}\right)$$

$$\mathcal{S} = \int d\tau \sum_{\mathbf{k}} c^\dagger_{\mathbf{k}\alpha} \left(\frac{\partial}{\partial \tau} - \varepsilon_{\mathbf{k}} \right) c_{\mathbf{k}\alpha}$$

$$- \lambda \int d\tau \sum_i c^\dagger_{i\alpha} \vec{\Phi}_i \cdot \vec{\sigma}_{\alpha\beta} c_{i\beta}$$

$$+ \quad V(\vec{\Phi})$$

We have exactly transformed the Hubbard model to the "spin-fermion" model with electronic Hamiltonian described by **electrons** $c_{i\alpha}$ on the square or triangular lattice with dispersion

$$\begin{aligned}\mathcal{H}_c &= -\sum_{i,\rho} t_\rho \left(c_{i,\alpha}^\dagger c_{i+\boldsymbol{v}_\rho,\alpha} + c_{i+\boldsymbol{v}_\rho,\alpha}^\dagger c_{i,\alpha} \right) \\ &\quad -\mu \sum_i c_{i,\alpha}^\dagger c_{i,\alpha} + \mathcal{H}_{\rm int}\end{aligned}$$

are coupled to a magnetic moment order parameter $\Phi^p(i)$, $p = x, y, z$

$$\mathcal{H}_{\rm int} = -\lambda \sum_i \Phi^p(i) c_{i,\alpha}^\dagger \sigma^p_{\alpha\beta} c_{i,\beta} + V_\Phi$$

Gauge theory of fluctuating antiferromagnetism

For fluctuating antiferromagnetism (spin density waves (SDW)), we transform to a rotating reference frame using the SU(2) rotation R_i

$$\left(\begin{array}{c} c_{i\uparrow} \\ c_{i\downarrow} \end{array} \right) = R_i \left(\begin{array}{c} \psi_{i,+} \\ \psi_{i,-} \end{array} \right),$$

in terms of fermionic "chargons" ψ_s and a **Higgs field** $H^a(i)$

$$\sigma^p \Phi^p(i) = R_i \, \sigma^a H^a(i) \, R_i^\dagger$$

The Higgs field is the SDW order in the rotating reference frame.
We will see later that the ψ_s are ϵ particles of the $\mathbb{Z}_2$ spin liquid.

Gauge theory of fluctuating antiferromagnetism

The SU(2) rotation R_i obeys $R_i^\dagger R = 1$ and so we write

$$R = \begin{pmatrix} z_\uparrow & -z_\downarrow^* \\ z_\downarrow & z_\uparrow^* \end{pmatrix}$$

The z_α are spin $S = 1/2$ bosonic spinons. We will see later that the z_α will become the e particles of the $\mathbb{Z}_2$ spin liquid.

Gauge theory of fluctuating antiferromagnetism

Field	Symbol	Statistics	$SU(2)_{\text{gauge}}$	$SU(2)_{\text{spin}}$	$U(1)_{\text{e.m.charge}}$
Electron	c	fermion	$\mathbf{1}$	$\mathbf{2}$	-1
AF order	Φ	boson	$\mathbf{1}$	$\mathbf{3}$	0
Chargon	ψ	fermion	$\mathbf{2}$	$\mathbf{1}$	-1
Spinon	R or z	boson	$\bar{\mathbf{2}}$	$\mathbf{2}$	0
Higgs	H	boson	$\mathbf{3}$	$\mathbf{1}$	0

Note that this representation is ambiguous up to a SU(2) gauge transformation, V_i

$$\begin{pmatrix} \psi_{i,+} \\ \psi_{i,-} \end{pmatrix} \rightarrow V_i \begin{pmatrix} \psi_{i,+} \\ \psi_{i,-} \end{pmatrix}$$

$$R_i \rightarrow R_i V_i^\dagger$$

$$\sigma^a H^a(i) \rightarrow V_i \, \sigma^b H^b(i) \, V_i^\dagger .$$

S. Sachdev, M.A. Metlitski, Y. Qi, and C. Xu, PRB **80**, 155129 (2009)

Gauge theory of fluctuating antiferromagnetism

The simplest effective Hamiltonian for the fermionic chargons is the same as that for the electrons, with the magnetic order replaced by the Higgs field.

$$\mathcal{H}_\psi = -\sum_{i,\rho} t_\rho \left(\psi_{i,s}^\dagger \psi_{i+\boldsymbol{v}_\rho, s} + \psi_{i+\boldsymbol{v}_\rho, s}^\dagger \psi_{i,s} \right) - \mu \sum_i \psi_{i,s}^\dagger \psi_{i,s} + \mathcal{H}_{\rm int}$$

$$\mathcal{H}_{\rm int} = -\lambda \sum_i H^a(i) \psi_{i,s}^\dagger \sigma^a_{ss'} \psi_{i,s'} + V_H$$

IF we can transform to a rotating reference frame in which $H^a(i) =$ a constant independent of time, **THEN** the ψ fermions in the presence of fluctuating magnetism will inherit the Fermi surfaces (if present) of the electrons in the presence of static magnetism.

For insulating spin liquids, we consider the case where the chargons are fully gapped, and there are no Fermi surfaces.

S. Sachdev, M. A. Metlitski, Y. Qi, and C. Xu, PRB **80**, 155129 (2009)

Gauge theory of fluctuating antiferromagnetism

Field	Symbol	Statistics	$SU(2)_{\rm gauge}$	$SU(2)_{\rm spin}$	$U(1)_{\rm e.m.charge}$
Electron	c	fermion	$\mathbf{1}$	$\mathbf{2}$	-1
AF order	Φ	boson	$\mathbf{1}$	$\mathbf{3}$	0
Chargon	ψ	fermion	$\mathbf{2}$	$\mathbf{1}$	-1
Spinon	R or z	boson	$\bar{\mathbf{2}}$	$\mathbf{2}$	0
Higgs	H	boson	$\mathbf{3}$	$\mathbf{1}$	0

The Higgs phases of the SU(2) gauge theory can realize states with topological order. The topological order depends upon the structure of the Higgs condensate.

Gauge theory of fluctuating antiferromagnetism

We obtain different numbers of adjoint Higgs scalars, N_h, depending upon the spatial dependence of the local spin correlations:

Neel correlations (un- and electron-doped cuprates):
$N_h = 1$,
$\boldsymbol{K} = (\pi, \pi)$,
$H^a(i) = H_1^a(\boldsymbol{r}) e^{i\boldsymbol{K}\cdot\boldsymbol{r}_i}$

Coplanar spin correlations on the triangular lattice :
$N_h = 2$,
$\boldsymbol{K} = (4\pi/3, 4\pi/\sqrt{3})$,
$H^a(i) = \text{Re}\left\{\left[H_1^a(\boldsymbol{r}) + iH_2^a(\boldsymbol{r})\right] e^{i\boldsymbol{K}_x\cdot\boldsymbol{r}_i}\right\}$

Bidirectional incommensurate correlations (hole doped cuprates):
$N_h = 4$,
$\boldsymbol{K}_y = (\pi, \pi - \delta)$, $\boldsymbol{K}_x = (\pi - \delta, \pi)$,
$H^a(i) = \text{Re}\left\{\left[H_1^a(\boldsymbol{r}) + iH_2^a(\boldsymbol{r})\right] e^{i\boldsymbol{K}_x\cdot\boldsymbol{r}_i} + \left[H_3^a(\boldsymbol{r}) + iH_4^a(\boldsymbol{r})\right] e^{i\boldsymbol{K}_y\cdot\boldsymbol{r}_i}\right\}$

Gauge theory of fluctuating antiferromagnetism

We obtain different numbers of adjoint Higgs scalars, N_h, depending upon the spatial dependence of the local spin correlations:

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Spin liquid on the triangular lattice

SU(2) gauge theory

For the triangular lattice, $N_h = 2$, we define the complex Higgs field

$$\mathcal{H}^a = H_1^a + i H_2^a .$$

The SU(2) gauge theory is

$$\mathcal{L} = \frac{1}{2} \left| \partial_\mu \mathcal{H}^a - \epsilon_{abc} A_\mu^b \mathcal{H}^c \right|^2 + \frac{1}{4g^2} F_{\mu\nu}^a F_{\mu\nu}^a + V(\mathcal{H}^a)$$

$$\begin{aligned} F_{\mu\nu}^a &= \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - \epsilon_{abc} A_\mu^b A_\nu^c \\ V(\mathcal{H}^a) &= s \left(\mathcal{H}^{a*} \mathcal{H}^a \right) + u_0 \left(\mathcal{H}^{a*} \mathcal{H}^a \right)^2 + u_1 \left| \mathcal{H}^a \mathcal{H}^a \right|^2 \\ &\quad + u_3 \left(\mathcal{H}^a \mathcal{H}^a \right)^3 + \text{c.c.} . \end{aligned}$$

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- For $u_1 > 0$, we obtain a Higgs phase $\langle \mathcal{H}^a \rangle \propto (1, i, 0)$
- This corresponds to fluctuating coplanar spin configurations of the triangular lattice.

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- The Higgs condensate breaks the SU(2) gauge symmetry down to a $\mathbb{Z}_2$ gauge symmetry (this is different from the Weinberg-Salam model): the condensate is only invariant a gauge transformation $\sigma^a H^a \rightarrow V \sigma^b H^b V^\dagger$ with $V = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$.

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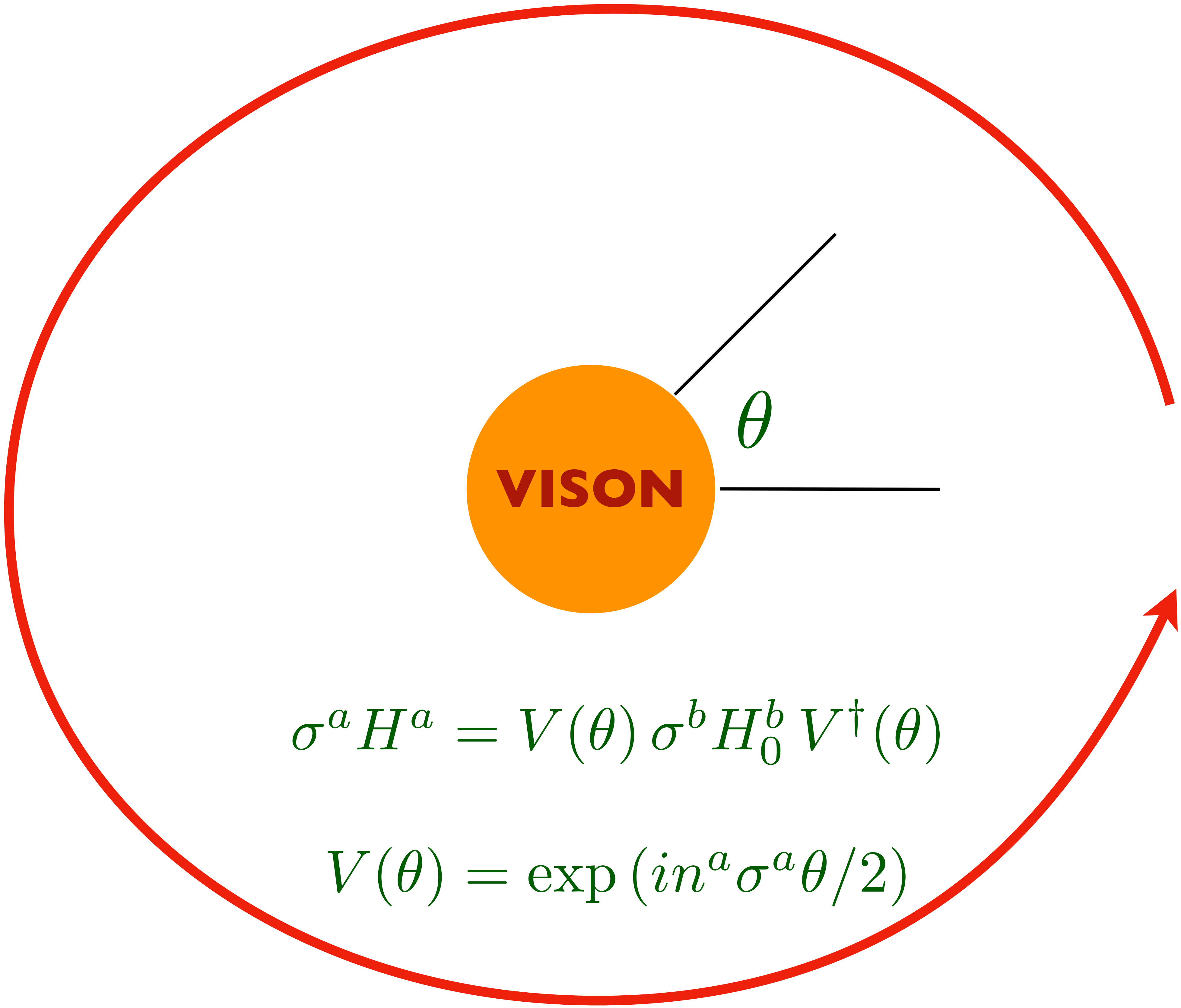
- The Higgs phase of the SU(2) gauge theory has finite energy $\mathbb{Z}_2$ vortex defects (visons!) associated with $\pi_1(SO(3)) = \mathbb{Z}_2$. These are analogous to $\mathbb{Z}$ Abrikosov vortices in the Ginzburg-Landau theory of superconductivity

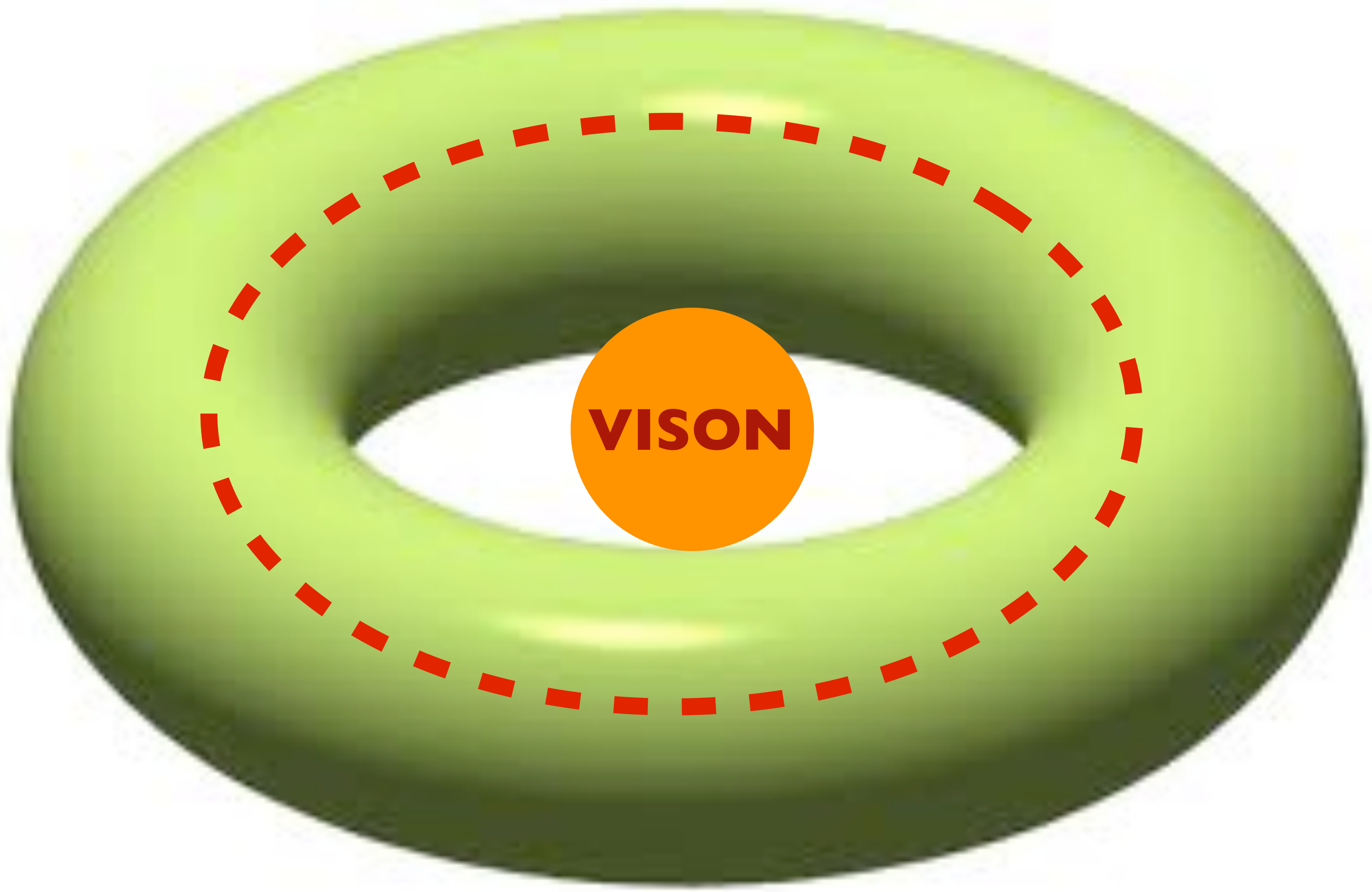
Spin liquid on the triangular lattice



$$\sigma^a H^a = V(\theta)\, \sigma^b H_0^b\, V^\dagger(\theta)$$

$$V(\theta) = \exp\left(i n^a \sigma^a \theta / 2\right)$$

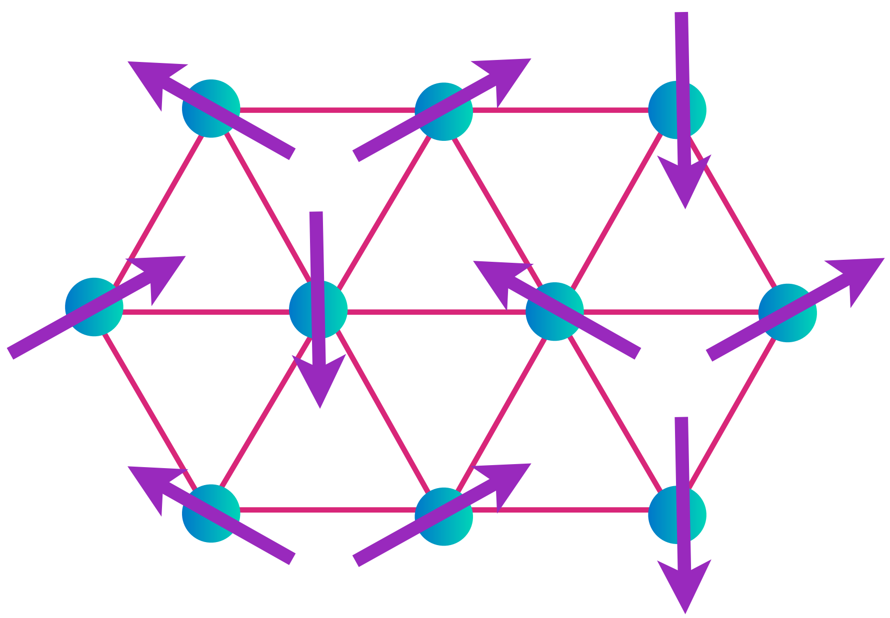
Spin liquid on the triangular lattice



4-fold degeneracy on the torus

Mott insulator: Triangular lattice antiferromagnet

Higgs condensate $\langle \mathcal{H}^a \rangle \propto (1, i, 0)$
Spinons R condensed $\langle R \rangle \neq 0$



non-collinear Néel state

Higgs condensate $\langle \mathcal{H}^a \rangle \propto (1, i, 0)$
Spinons R gapped $\langle R \rangle = 0$

Z_2 spin liquid
with neutral $S = 1/2$ spinons
and **vison** excitations

s_c

s

$O(4)^*$ CFT3

A.V. Chubukov, T. Senthil and S. Sachdev, Physical Review Letters **72**, 2089 (1994)

Mott insulator: Triangular lattice antiferromagnet

Field	Symbol	Statistics	$SU(2)_{\text{gauge}}$	$SU(2)_{\text{spin}}$	$U(1)_{\text{e.m.charge}}$	$\mathbb{Z}_2$ type
Electron	c	fermion	$\mathbf{1}$	$\mathbf{2}$	-1	1
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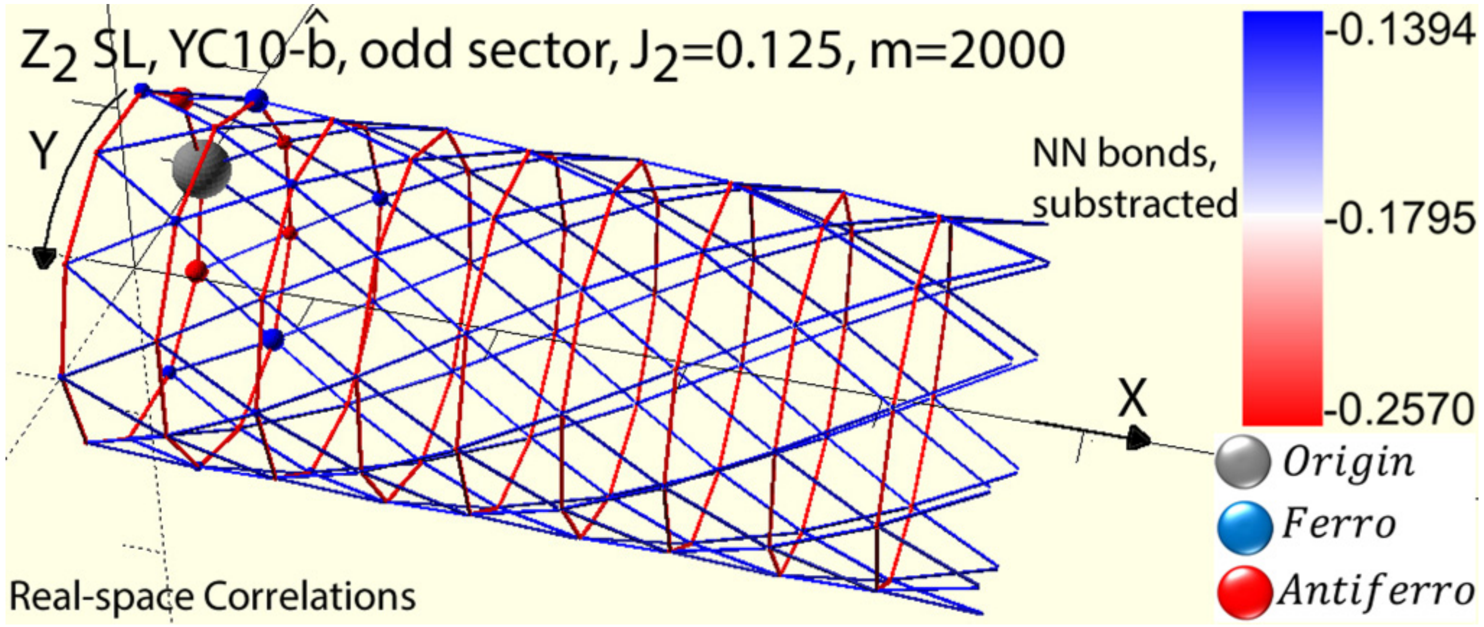
Chargon	ψ	fermion	$\mathbf{2}$	$\mathbf{1}$	-1	ϵ
Spinon	R or z	boson	$\bar{\mathbf{2}}$	$\mathbf{2}$	0	e
Higgs	H	boson	$\mathbf{3}$	$\mathbf{1}$	0	1
Vison	m	boson	$\mathbf{1}$	$\mathbf{1}$	0	m

Symmetry fractionalization in the topological phase of the spin-$\frac{1}{2}$ J_1-J_2 triangular Heisenberg model

S. N. Saadatmand* and I. P. McCulloch
ARC Centre for Engineered Quantum Systems, School of Mathematics and Physics,
The University of Queensland, St. Lucia, Queensland 4072, Australia
(Received 15 July 2016; published 13 September 2016)

Using density-matrix renormalization-group calculations for infinite cylinders, we elucidate the properties of the spin-liquid phase of the spin-$\frac{1}{2}$ J_1-J_2 Heisenberg model on the triangular lattice. We find *four* distinct ground states characteristic of a nonchiral, Z_2 topologically ordered state with vison and spinon excitations. We shed light on the interplay of topological ordering and global symmetries in the model by detecting fractionalization of time-reversal and space-group dihedral symmetries in the anyonic sectors, which leads to the coexistence of symmetry protected and intrinsic topological order. The anyonic sectors, and information on the particle statistics, can be characterized by degeneracy patterns and symmetries of the entanglement spectrum. We demonstrate the ground states on finite-width cylinders are short-range correlated and gapped; however, some features in the entanglement spectrum suggest that the system develops gapless spinonlike edge excitations in the large-width limit.

PHYSICAL REVIEW B **94**, 121111(R) (2016)



1. Resonating valence bonds

The Z_2 spin liquid

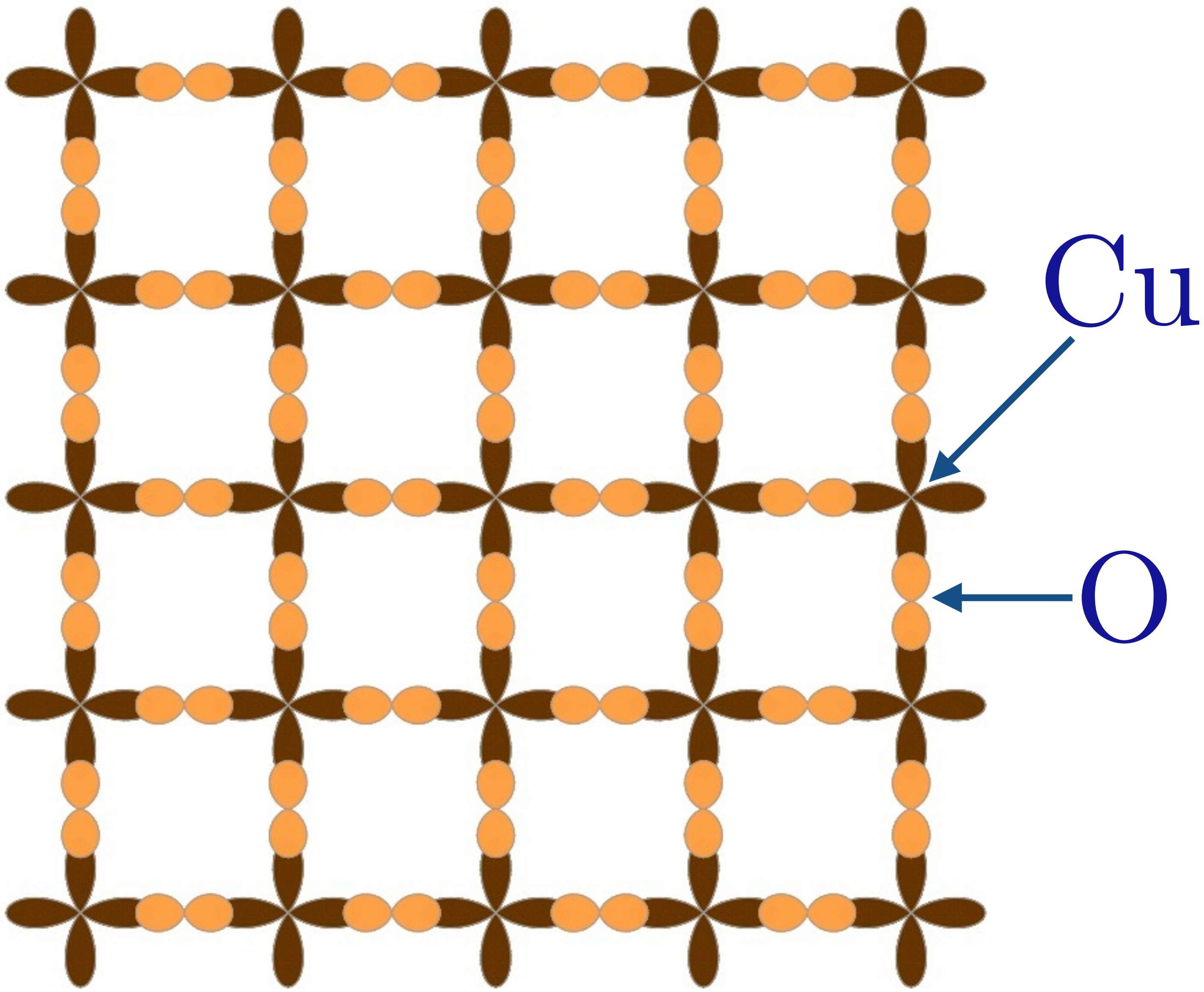
2. SU(2) gauge theory of fluctuating antiferromagnetism on the triangular lattice

The Z_2 spin liquid

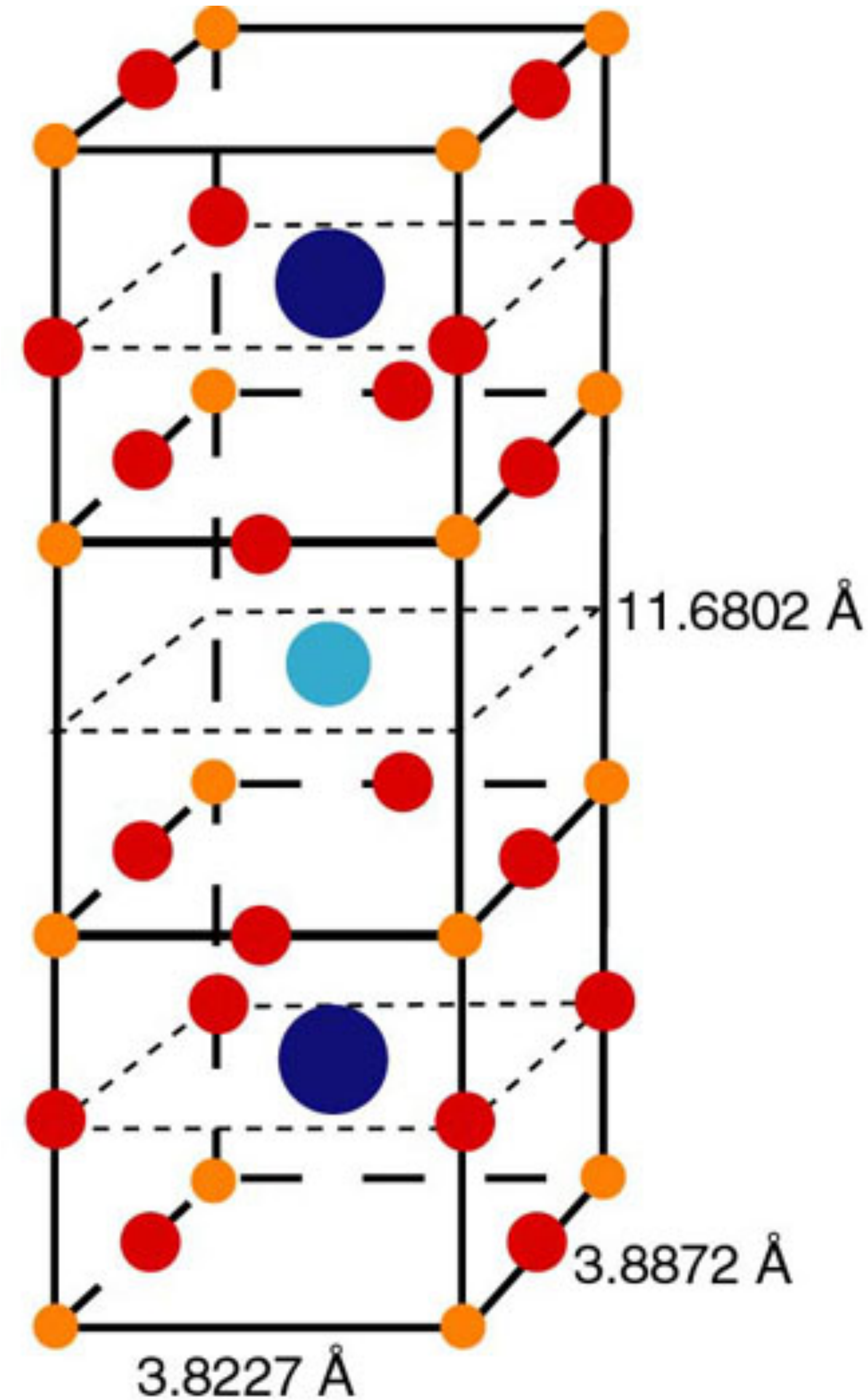
3. Electron-doped cuprates

Higgs phase with topological order: Fermi surface reconstruction without translational symmetry breaking

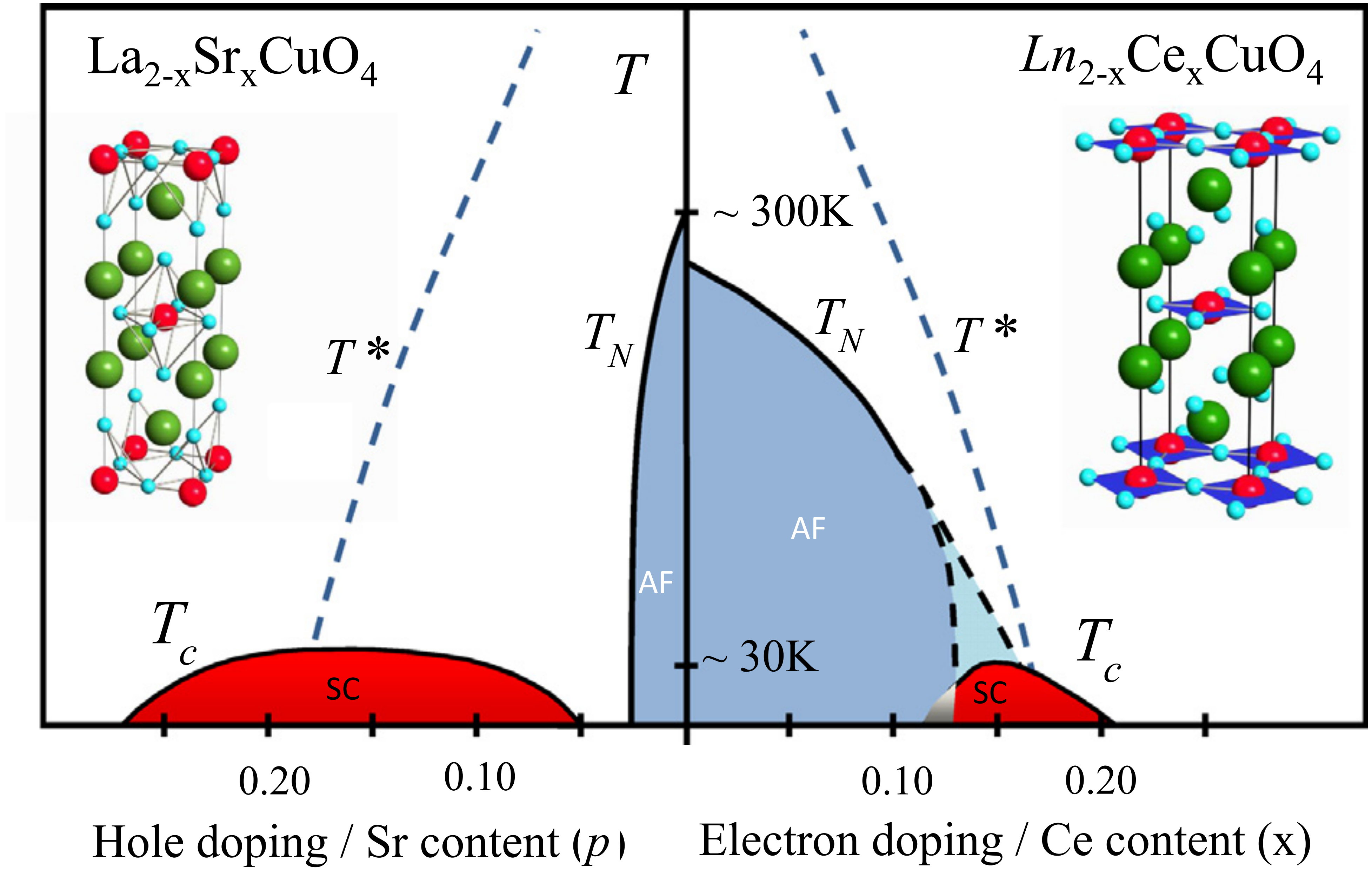
High temperature superconductors



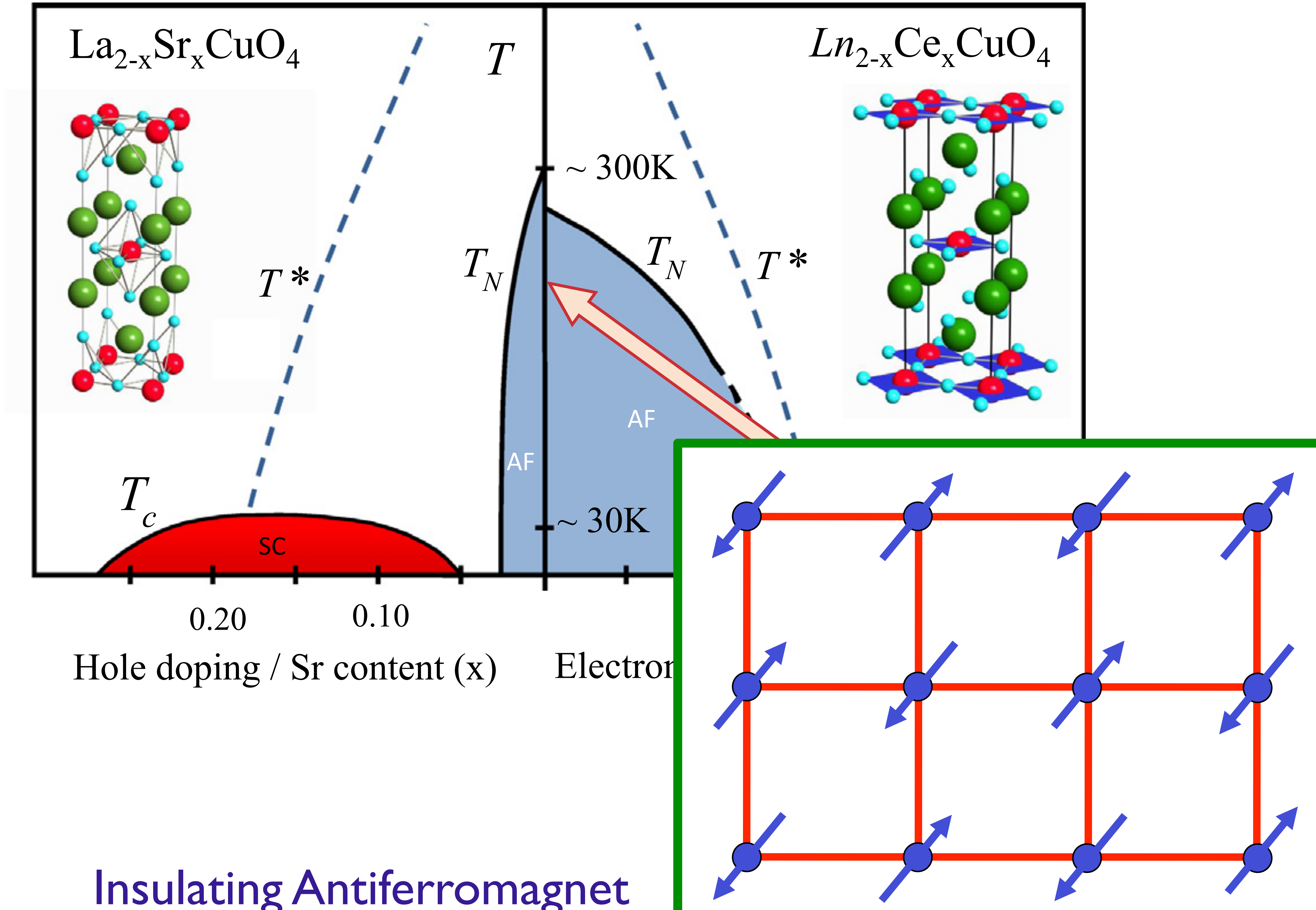
CuO_2 plane



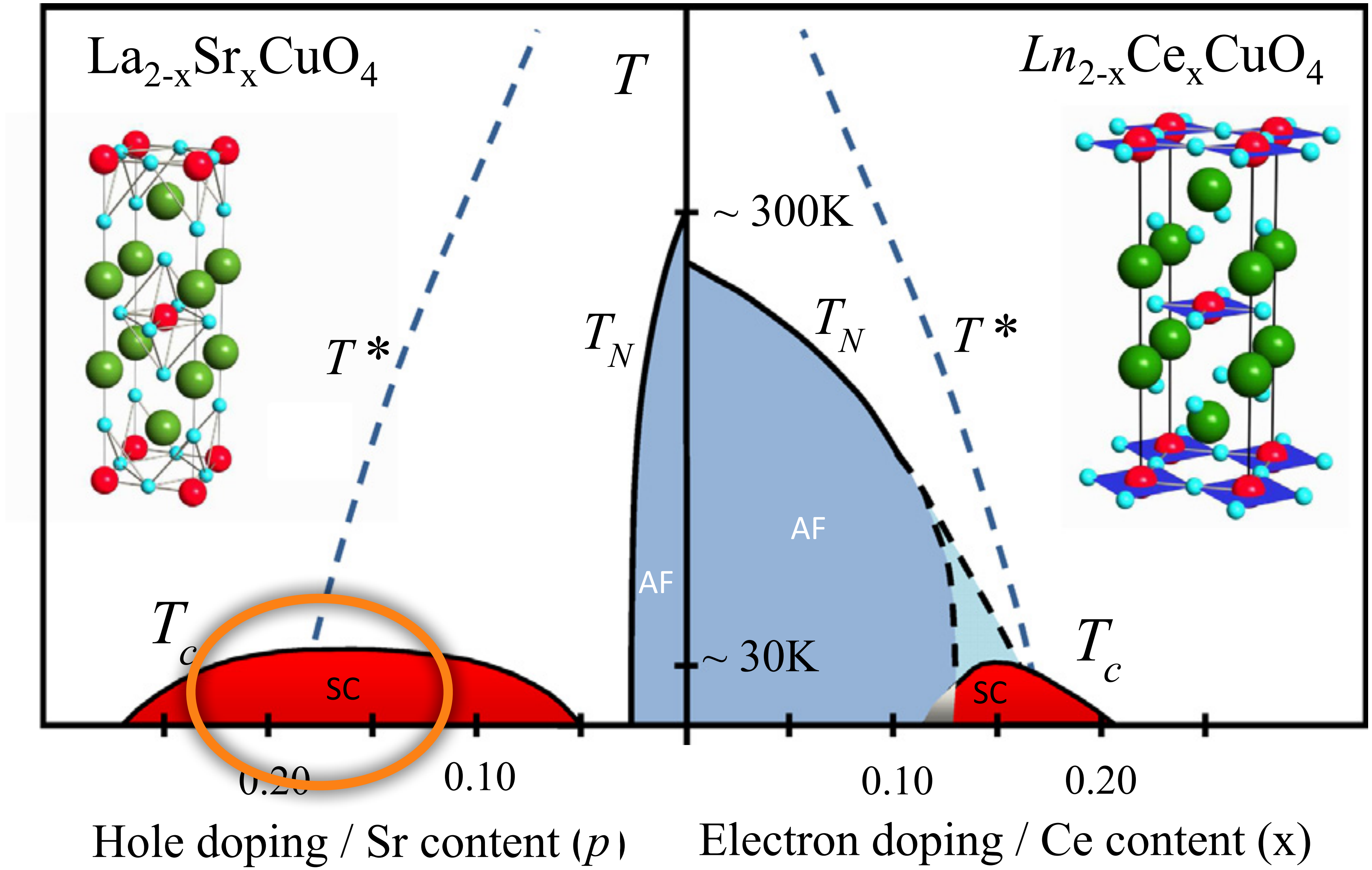
$YBa_2Cu_3O_{6+x}$



$La_{2-x}Sr_xCuO_4$
T
$Ln_{2-x}Ce_xCuO_4$
~ 300K
T_N
T_N
T^*
T^*
AF
AF
T_c
T_c
~ 30K
SC
SC
0.20
0.10
0.10
0.20
Hole doping / Sr content (*p*)
Electron doping / Ce content (x)



Insulating Antiferromagnet



$La_{2-x}Sr_xCuO_4$
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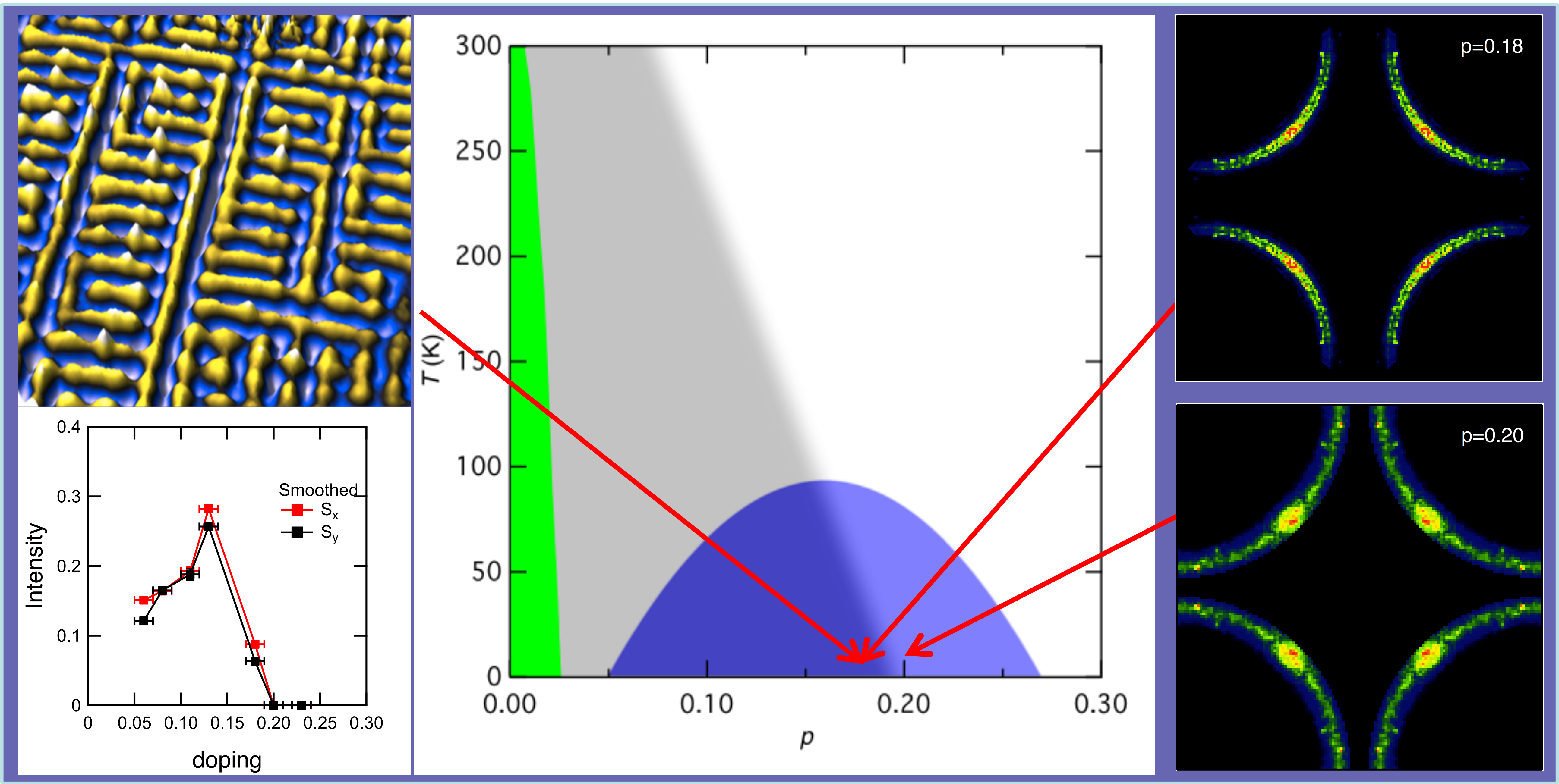
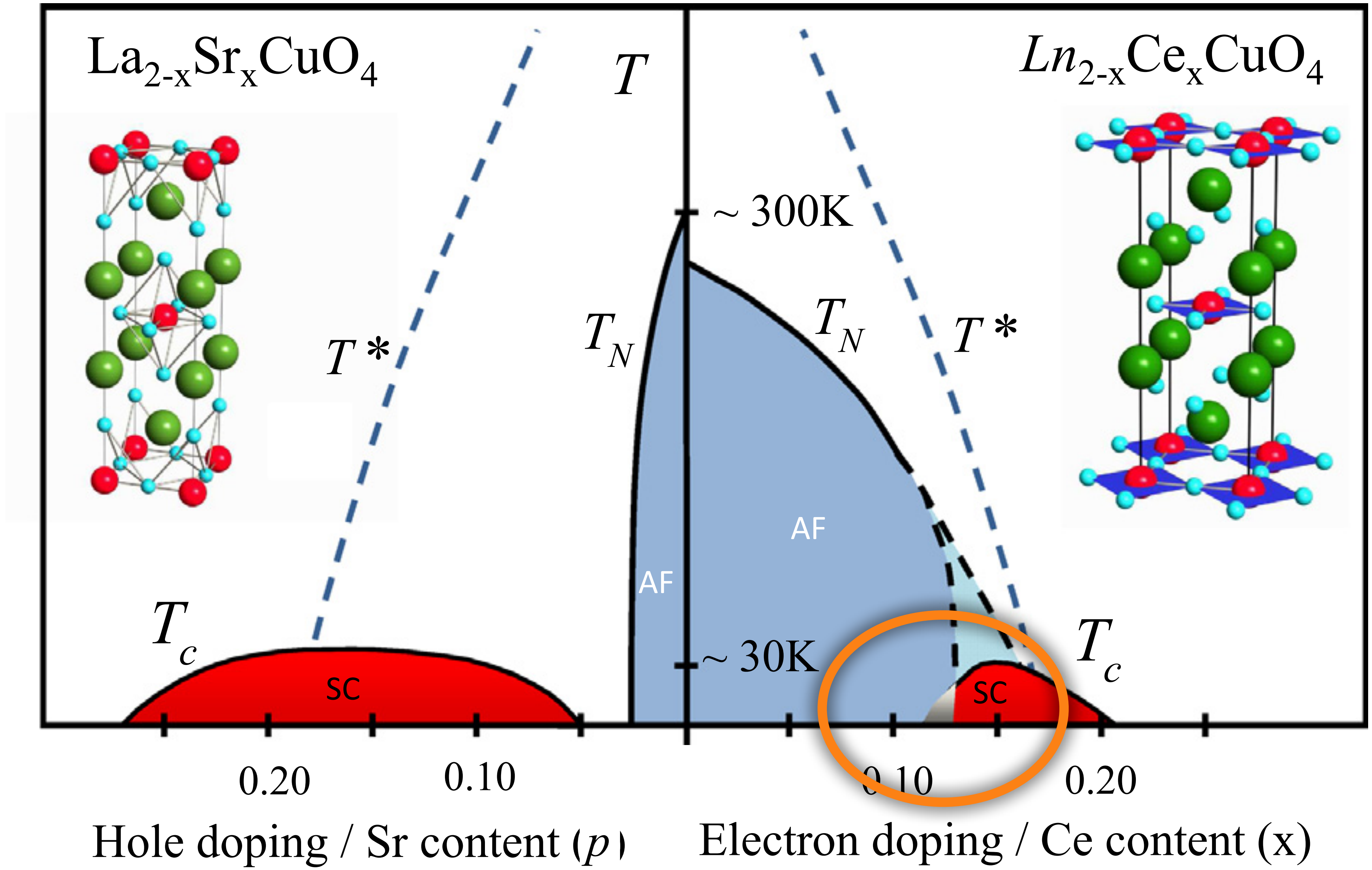
Hole doping / Sr content (p)
Electron doping / Ce content (x)

Hole doped cuprates

Yang He, Yi Yin, M. Zech, A. Soumyanarayanan, I. Zeljkovic, M. M. Yee, M. C. Boyer, K. Chatterjee, W. D. Wise, Takeshi Kondo, T. Takeuchi, H. Ikuta, P. Mistark, R. S. Markiewicz, A. Bansil, S. Sachdev, E. W. Hudson, and J. E. Hoffman, Science **344**, 608 (2014)

K. Fujita, Chung Koo Kim, Inhee Lee, Jinho Lee, M. H. Hamidian, I. A. Firmo, S. Mukhopadhyay, H. Eisaki, S. Uchida, M. J. Lawler, E.-A. Kim, J. C. Davis, Science **344**, 612 (2014)





$La_{2-x}Sr_xCuO_4$
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~ 300K
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SC
0.20
0.10
0.10
0.20
Hole doping / Sr content (p)
Electron doping / Ce content (x)

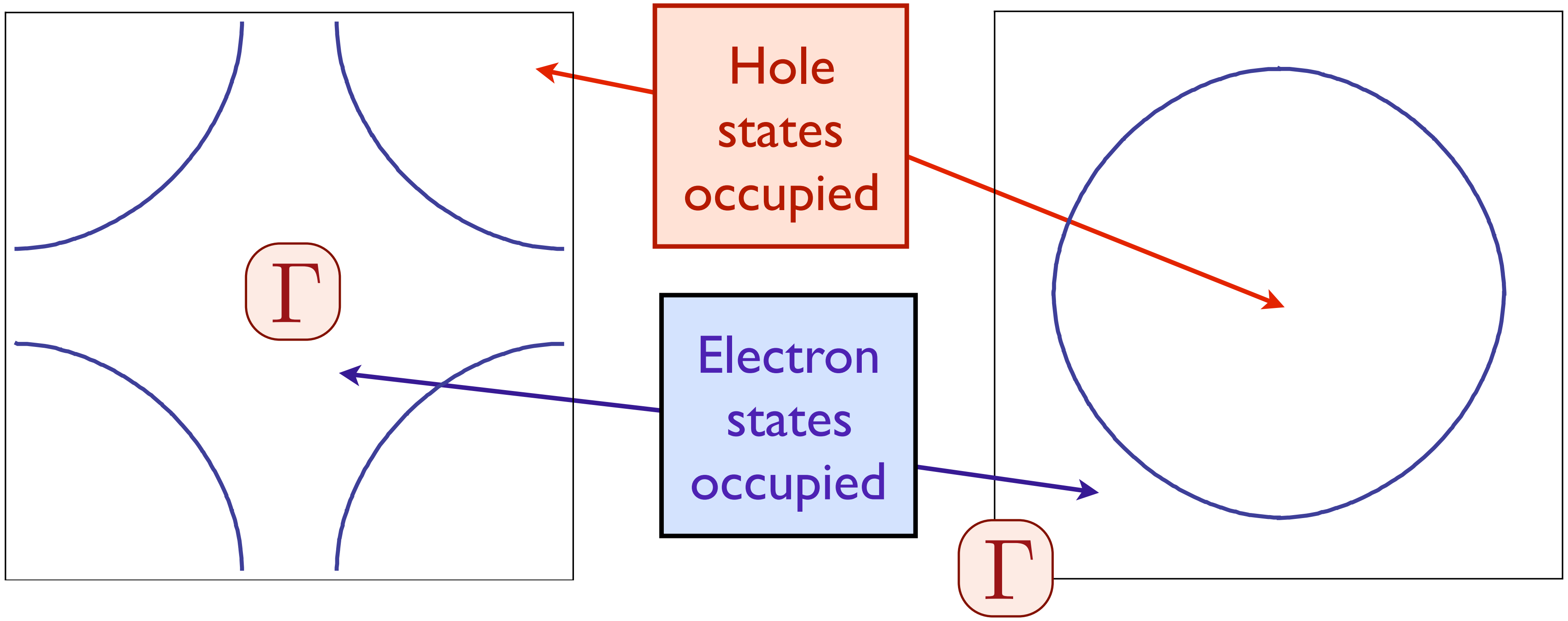
We have exactly transformed the Hubbard model to the "spin-fermion" model with electronic Hamiltonian described by **electrons** $c_{i\alpha}$ on the square or triangular lattice with dispersion

$$
\begin{aligned}
\mathcal{H}_c &= -\sum_{i,\rho} t_\rho \left(c_{i,\alpha}^\dagger c_{i+\boldsymbol{v}_\rho,\alpha} + c_{i+\boldsymbol{v}_\rho,\alpha}^\dagger c_{i,\alpha} \right) \\
&\quad -\mu \sum_i c_{i,\alpha}^\dagger c_{i,\alpha} + \mathcal{H}_{\text{int}}
\end{aligned}
$$

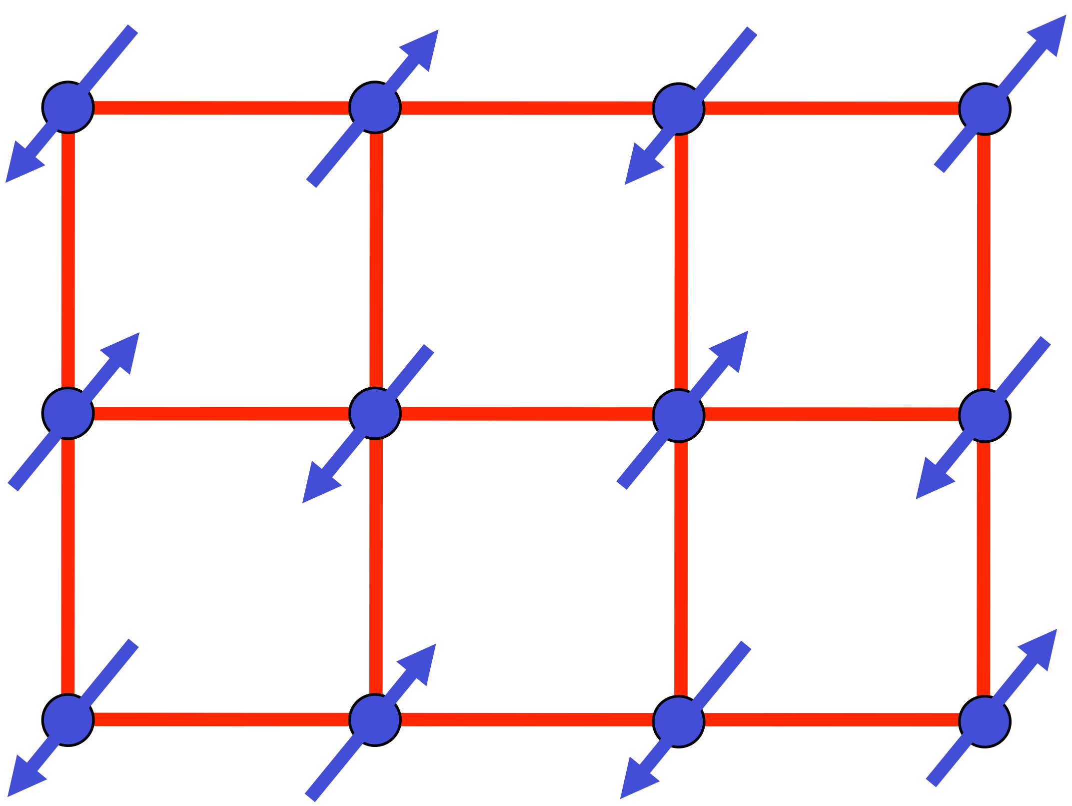
are coupled to a magnetic moment order parameter $\Phi^p(i)$, $p = x, y, z$

$$
\mathcal{H}_{\text{int}} = -\lambda \sum_i \Phi^p(i) c_{i,\alpha}^\dagger \sigma_{\alpha\beta}^p c_{i,\beta} + V_\Phi
$$

Fermi surface+antiferromagnetism



+



The electron spin polarization obeys

$$\left\langle \vec{\Phi}(\mathbf{r},\tau) \right\rangle = \vec{\mathcal{N}} e^{i\mathbf{K}\cdot\mathbf{r}}$$

where $\mathbf{K} = (\pi,\pi)$ is the ordering wavevector.

Fermi surface+antiferromagnetism

In momentum space, the coupling between $\vec{\mathcal{N}}$ and the electrons takes the form

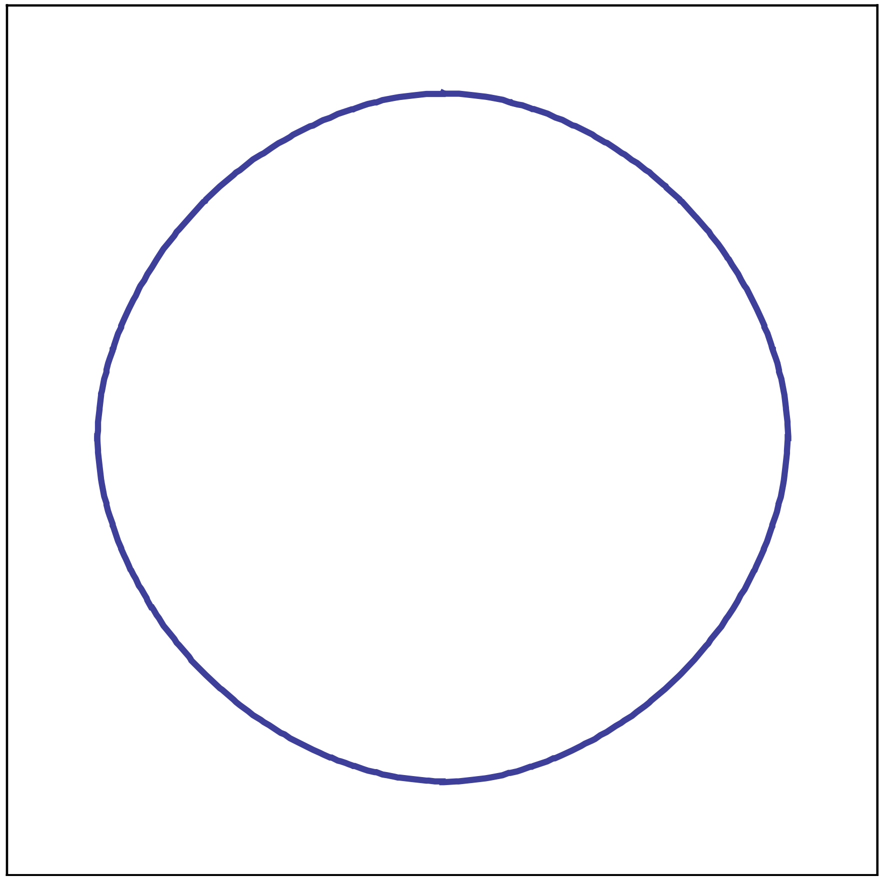
$$\mathcal{H}_{\text{int}} = \lambda \sum_{\mathbf{k},\mathbf{q},\alpha,\beta} \vec{\mathcal{N}}_{\mathbf{q}} \cdot c^{\dagger}_{\mathbf{k}+\mathbf{q},\alpha} \vec{\sigma}_{\alpha\beta} c_{\mathbf{k}+\mathbf{K},\beta}$$

where $\vec{\sigma}$ are the Pauli matrices, the boson momentum $\mathbf{q}$ is small, while the fermion momenum $\mathbf{k}$ extends over the entire Brillouin zone. In the antiferromagnetically ordered state, we may take $\vec{\mathcal{N}} \propto (0,0,1)$, and the electron dispersions obtained by diagonalizing $\mathcal{H}_c + \mathcal{H}_{\text{int}}$ are

$$E_{\mathbf{k}\pm} = \frac{\varepsilon_{\mathbf{k}} + \varepsilon_{\mathbf{k}+\mathbf{K}}}{2} \pm \sqrt{\left(\frac{\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k}+\mathbf{K}}}{2}\right)^2 + \lambda^2 |\vec{\mathcal{N}}|^2}$$

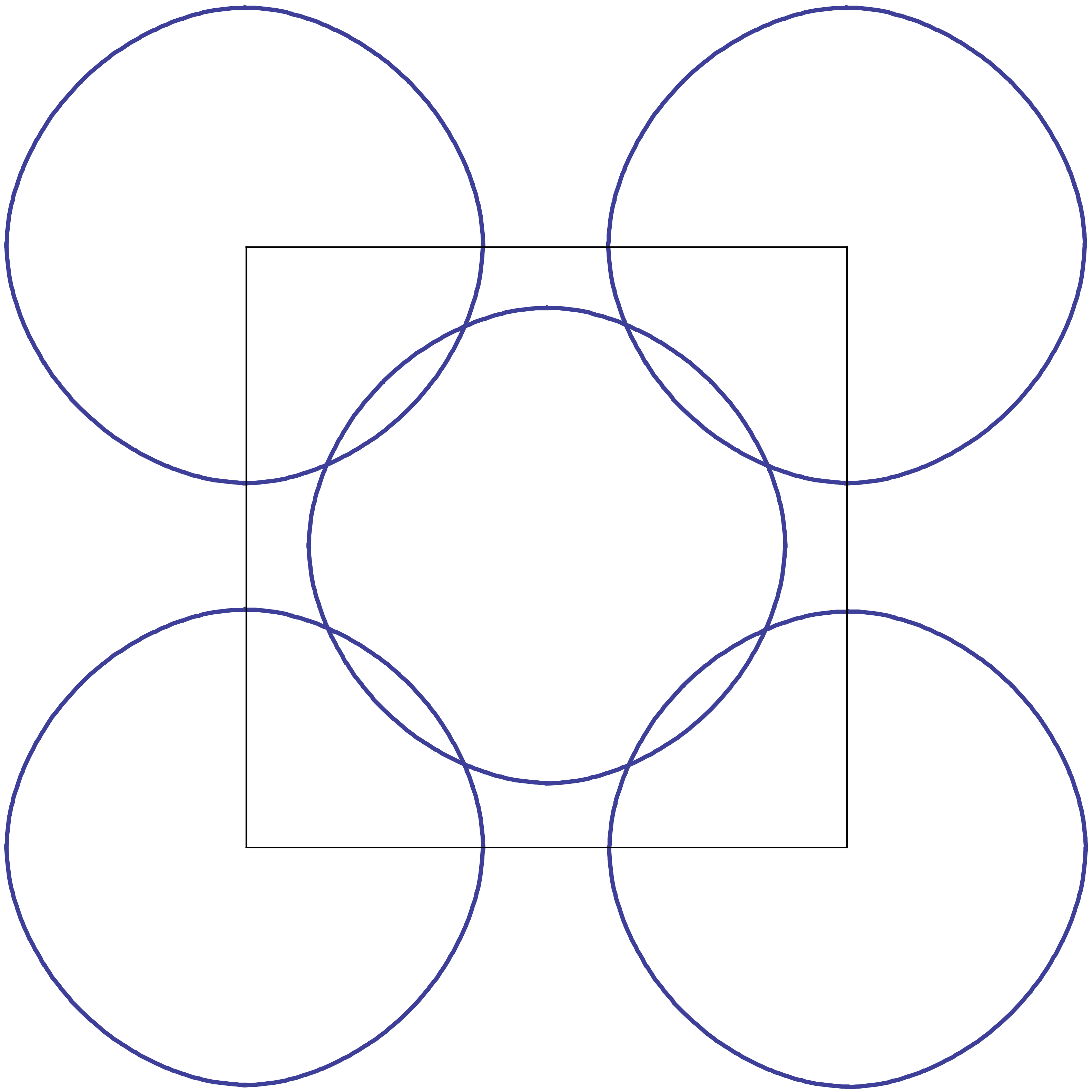
This leads to the Fermi surfaces shown in the following slides as a function of increasing $|\vec{\mathcal{N}}|$.

Fermi surface+antiferromagnetism



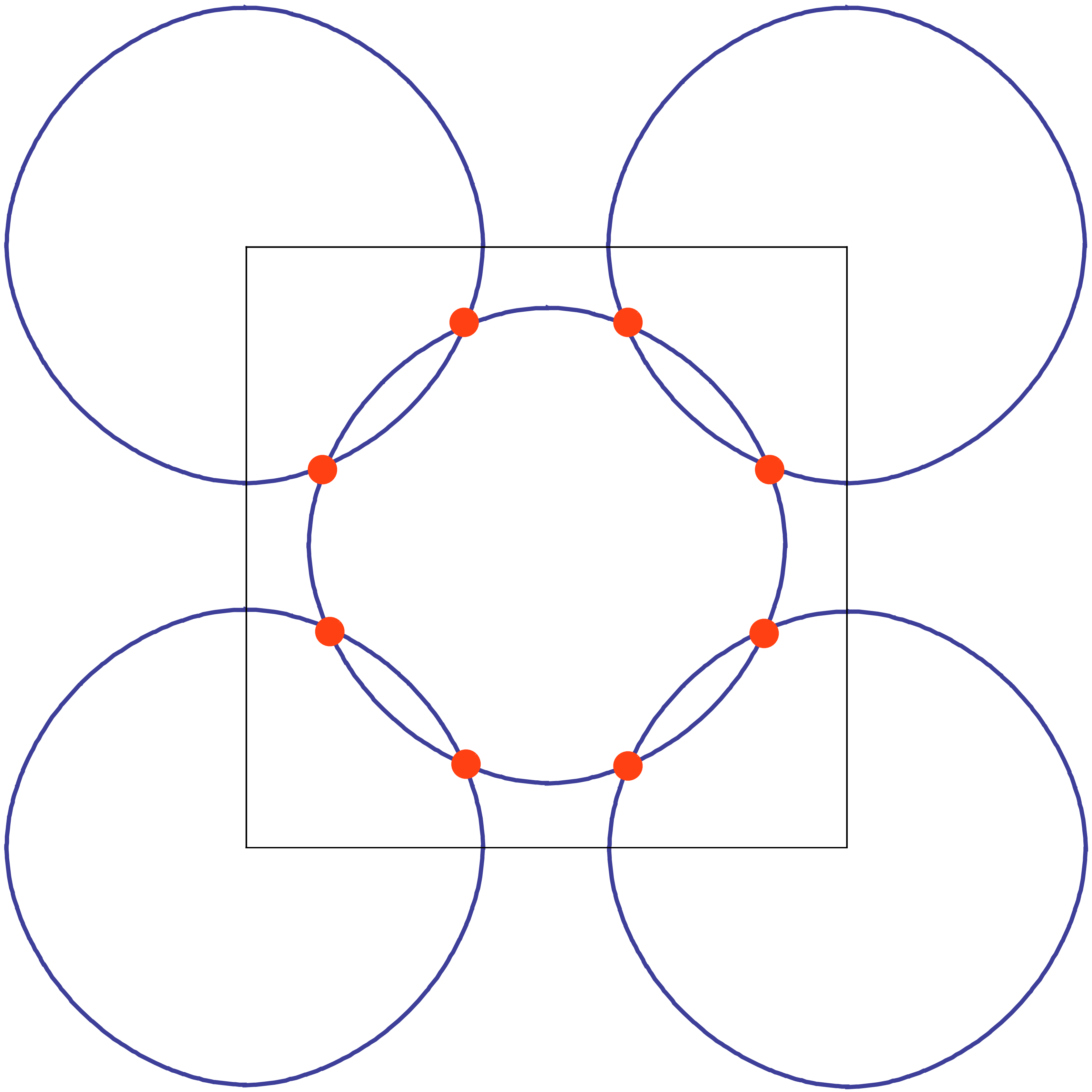
Metal with "large" Fermi surface

Fermi surface+antiferromagnetism



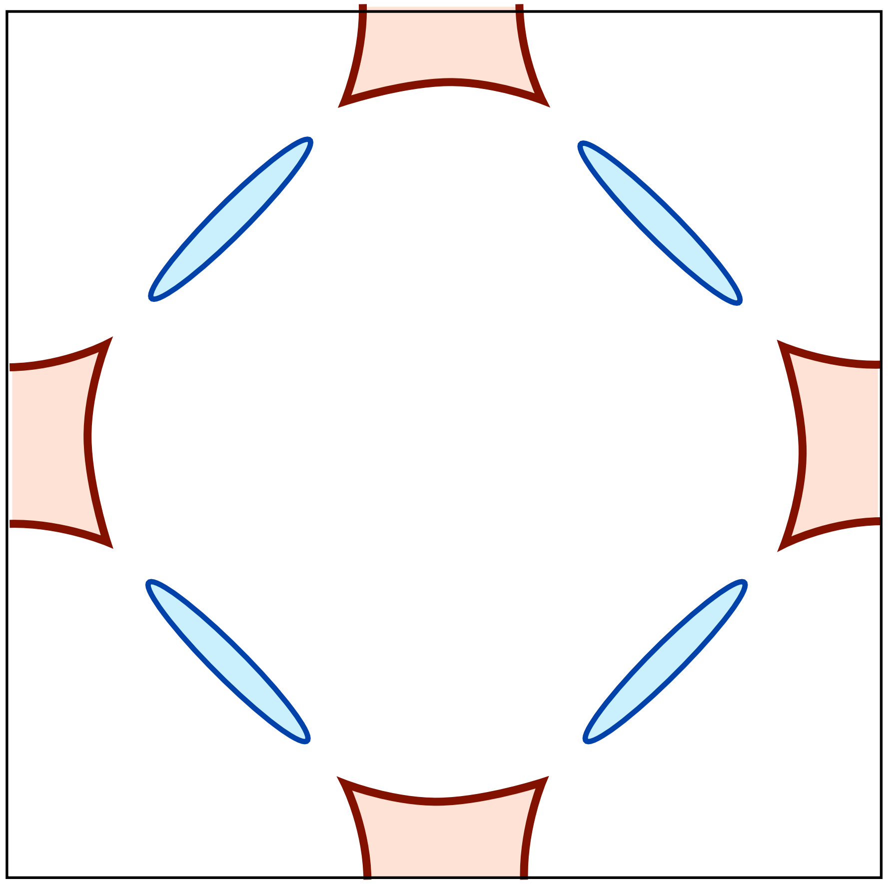
Fermi surfaces translated by $\mathbf{K} = (\pi, \pi)$.

Fermi surface+antiferromagnetism



"Hot" spots

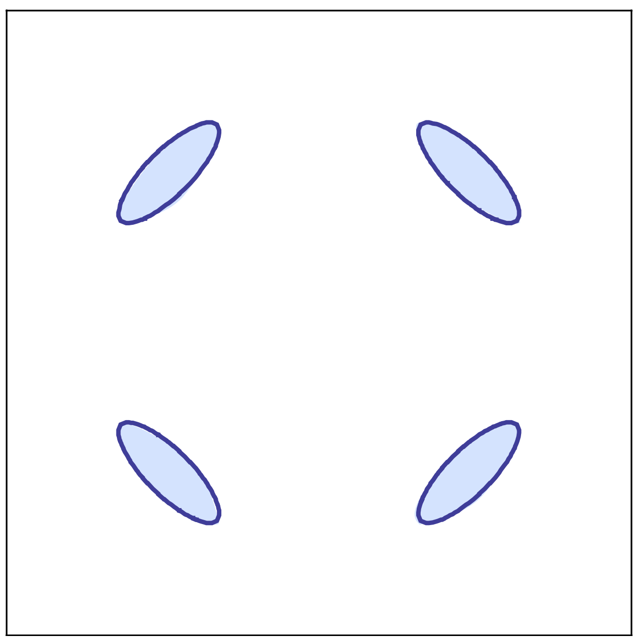
Fermi surface+antiferromagnetism



Electron and hole pockets in antiferromagnetic phase with $\langle \vec{\Phi} \rangle \neq 0$

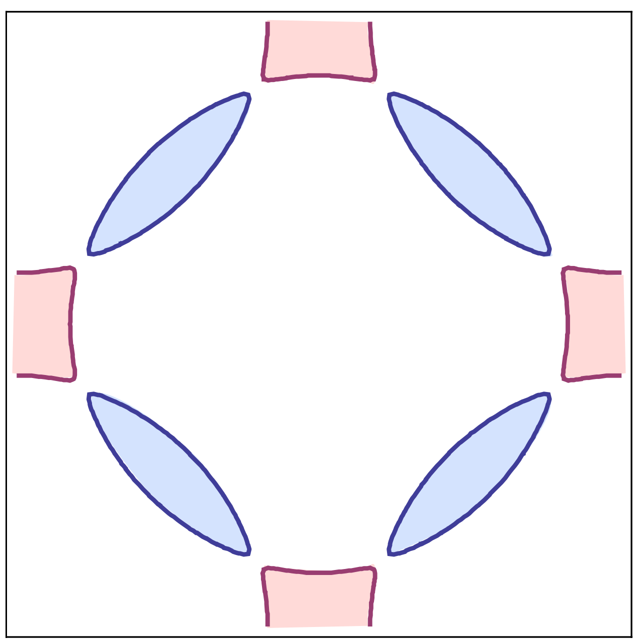
Square lattice Hubbard model with hole doping

$\langle \vec{\Phi} \rangle \neq 0$
and large



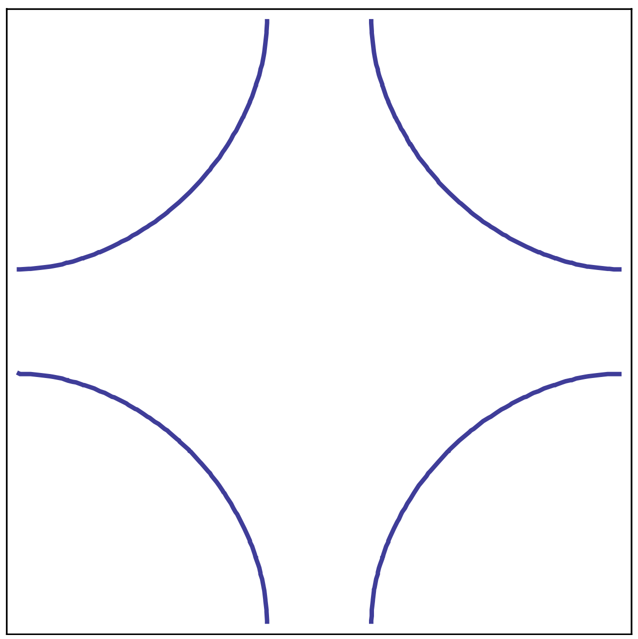
Metal with hole pockets

$\langle \vec{\Phi} \rangle \neq 0$
and small



Metal with electron and hole pockets

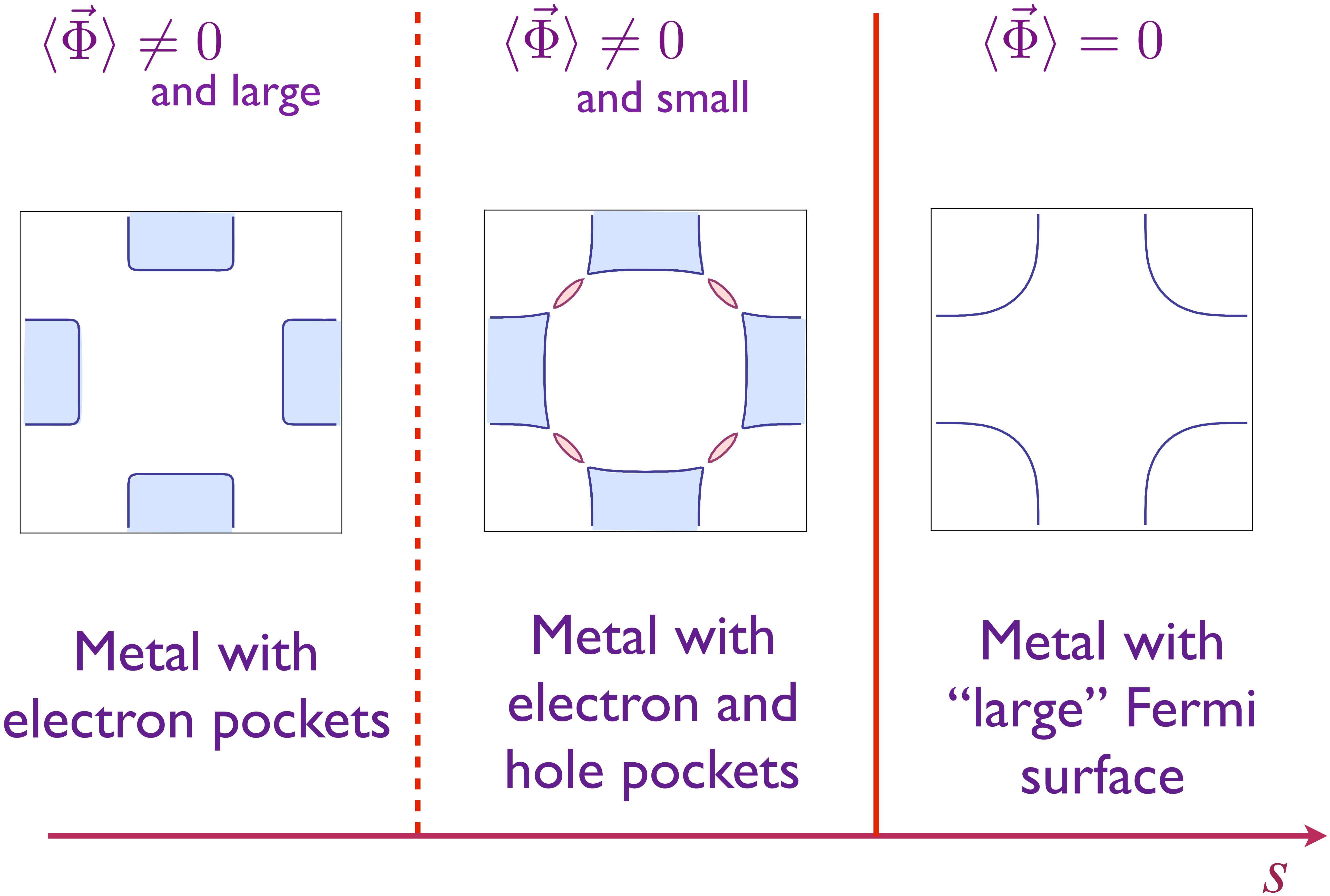
$\langle \vec{\Phi} \rangle = 0$



Metal with "large" Fermi surface

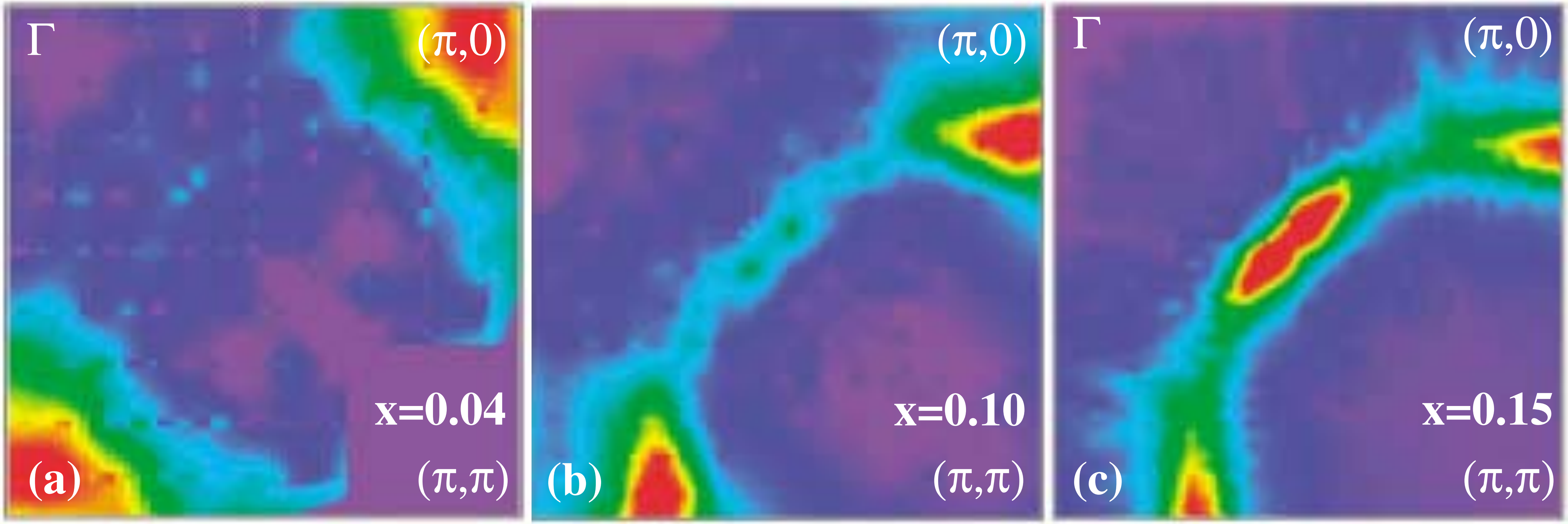
s

Square lattice Hubbard model with electron doping



Electron doped cuprates

$Nd_{2-x}Ce_xCuO_{4\pm\delta}$



Doping Dependence of an n-Type Cuprate Superconductor Investigated by Angle-Resolved Photoemission Spectroscopy

N. P. Armitage, F. Ronning, D. H. Lu, C. Kim, A. Damascelli, K. M. Shen, D. L. Feng, H. Eisaki, Z.-X. Shen, P. K. Mang, N. Kaneko, M. Greven, Y. Onose, Y. Taguchi, and Y. Tokura
Phys. Rev. Lett. **88**, 257001 (2002)

PNAS 116, 3449 (2019)

Fermi surface reconstruction in electron-doped cuprates without antiferromagnetic long-range order

Junfeng He, C. R. Rotundu, M. S. Scheurer, Y. He, M. Hashimoto, K. Xu, Y. Wang, E. W. Huang, T. Jia, S.-D. Chen, B. Moritz, D.-H. Lu, Y. S. Lee, T. P. Devereaux and Z.-X. Shen

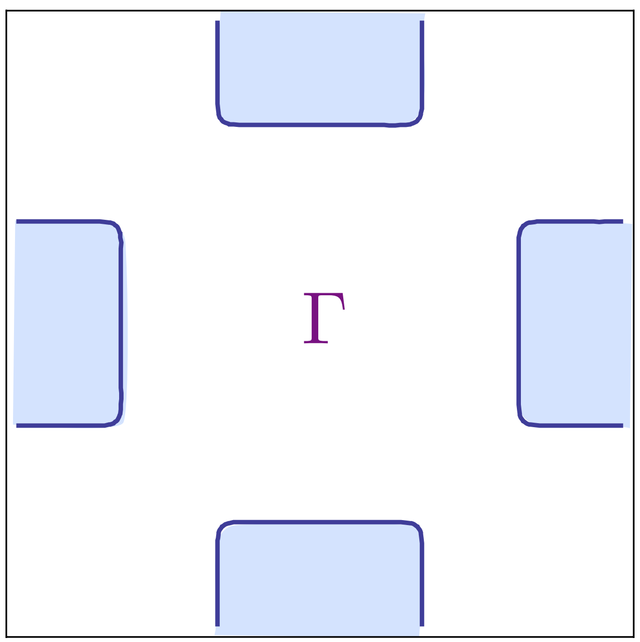
- New photoemission measurements at zero magnetic field show Fermi surfaces in quantitative agreement with quantum oscillation measurements.

- The energy gap between the electron and hole pockets collapses near $x = 0.17$ like an order parameter.

- "The totality of the data points to a mysterious order between $x = 0.14$ and $x = 0.17$, whose appearance favors the FS reconstruction and disappearance defines the quantum critical doping. A recent topological proposal provides an ansatz for its origin."

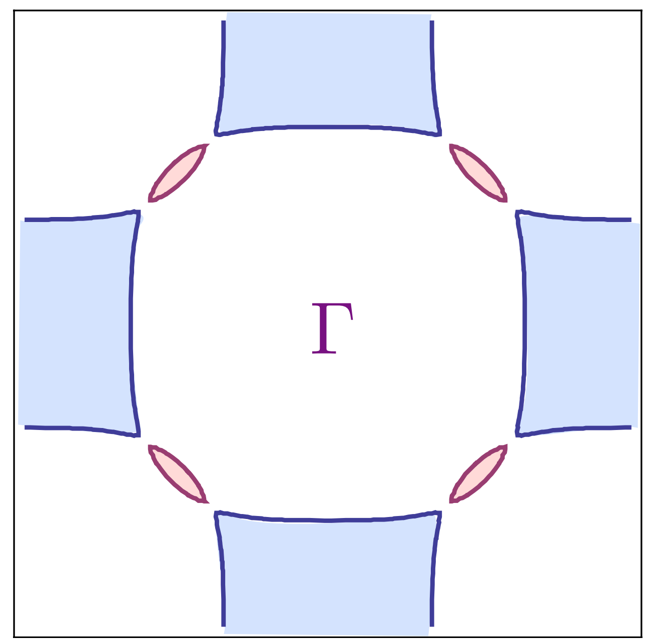
Square lattice Hubbard model with electron doping

$\langle \Phi^a \rangle \neq 0$ and large



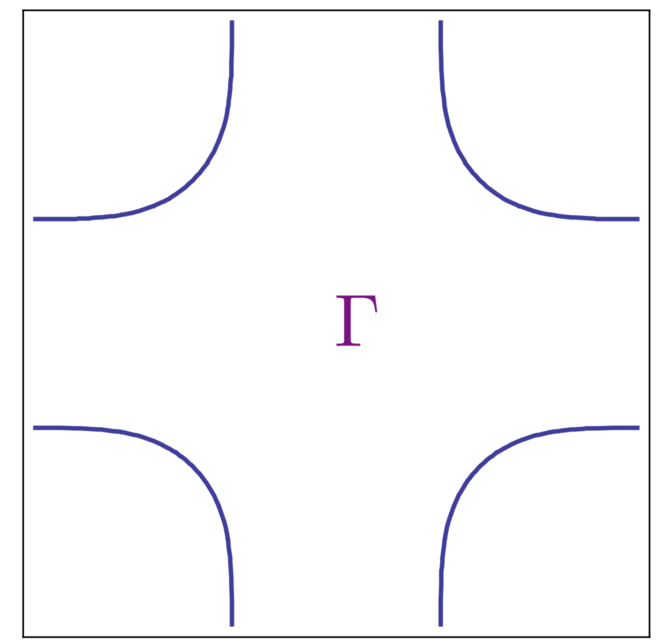
Metal with electron pockets

$\langle \Phi^a \rangle \neq 0$ and small



Metal with electron and hole pockets

$\langle \Phi^a \rangle = 0$

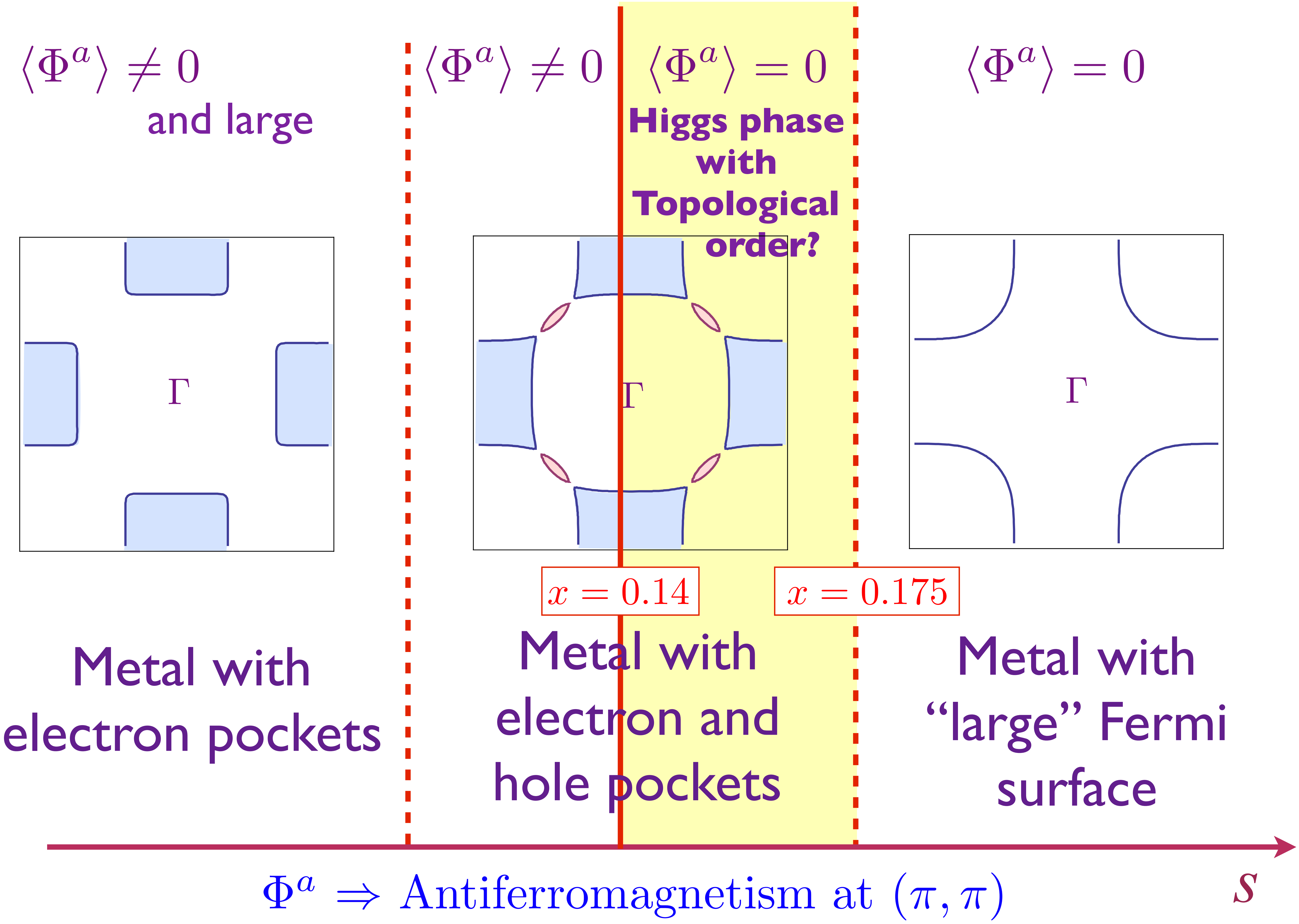


Metal with "large" Fermi surface

s

$\Phi^a \Rightarrow$ Antiferromagnetism at (π, π)

Square lattice Hubbard model with electron doping



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$$\mathcal{H}_\psi = -\sum_{i,\rho} t_\rho \left(\psi^\dagger_{i,s} \psi_{i+\boldsymbol{v}_\rho,s} + \psi^\dagger_{i+\boldsymbol{v}_\rho,s} \psi_{i,s} \right) - \mu \sum_i \psi^\dagger_{i,s} \psi_{i,s} + \mathcal{H}_{\rm int}$$

$$\mathcal{H}_{\rm int} = -\lambda \sum_i H^a(i) \psi^\dagger_{i,s} \sigma^a_{ss'} \psi_{i,s'} + V_H$$

IF we can transform to a rotating reference frame in which $H^a(i) =$ a constant independent of time, **THEN** the ψ fermions in the presence of fluctuating magnetism will inherit the Fermi surfaces (if present) of the electrons in the presence of static magnetism. For the electron-doped cuprates, the chargons acquire the small, reconstructed Fermi surfaces of the doped antiferromagnet.

S. Sachdev, M. A. Metlitski, Y. Qi, and C. Xu, PRB **80**, 155129 (2009)

Gauge theory of fluctuating antiferromagnetism

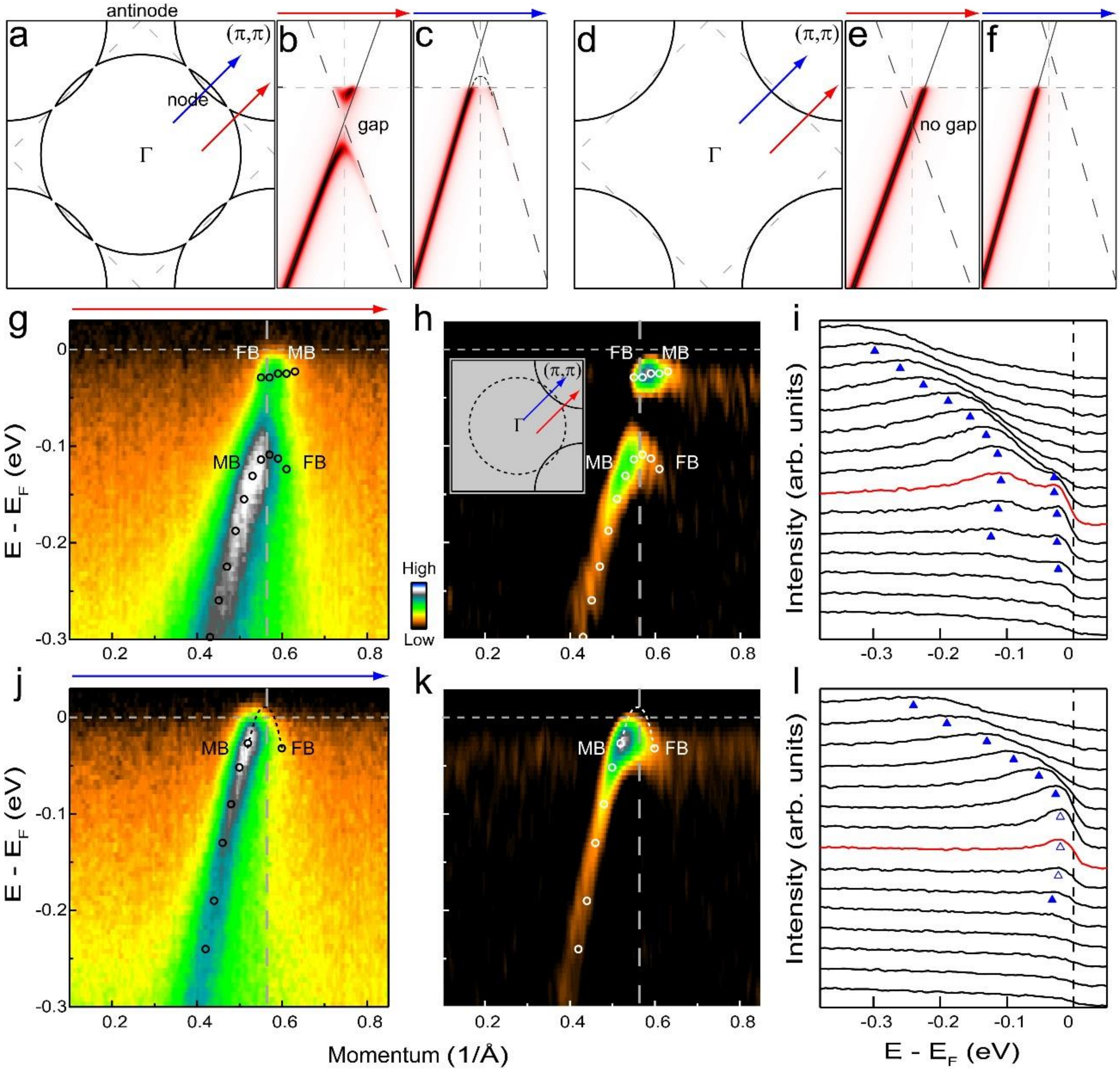
We obtain different numbers of adjoint Higgs scalars, N_h, depending upon the spatial dependence of the local spin correlations:

Neel correlations (un- and electron-doped cuprates):
$N_h = 1$,
$\boldsymbol{K} = (\pi, \pi)$,
$H^a(i) = H_1^a(\boldsymbol{r}) e^{i\boldsymbol{K}\cdot\boldsymbol{r}_i}$

SU(2) gauge symmetry broken down to U(1)

Coplanar spin correlations on the triangular lattice :
$N_h = 2$,
$\boldsymbol{K} = (4\pi/3, 4\pi/\sqrt{3})$,
$H^a(i) = \text{Re}\left\{\left[H_1^a(\boldsymbol{r}) + iH_2^a(\boldsymbol{r})\right] e^{i\boldsymbol{K}_x\cdot\boldsymbol{r}_i}\right\}$

Bidirectional incommensurate correlations (hole doped cuprates):
$N_h = 4$,
$\boldsymbol{K}_y = (\pi, \pi - \delta)$, $\boldsymbol{K}_x = (\pi - \delta, \pi)$,
$H^a(i) = \text{Re}\left\{\left[H_1^a(\boldsymbol{r}) + iH_2^a(\boldsymbol{r})\right] e^{i\boldsymbol{K}_x\cdot\boldsymbol{r}_i} + \left[H_3^a(\boldsymbol{r}) + iH_4^a(\boldsymbol{r})\right] e^{i\boldsymbol{K}_y\cdot\boldsymbol{r}_i}\right\}$



Junfeng He, C. R. Rotundu, M. S. Scheurer, Y. He, M. Hashimoto, K. Xu, Y. Wang, E. W. Huang, T. Jia, S.-D. Chen, B. Moritz, D.-H. Lu, Y. S. Lee, T. P. Devereaux and Z.-X. Shen PNAS **116**, 3449 (2019)

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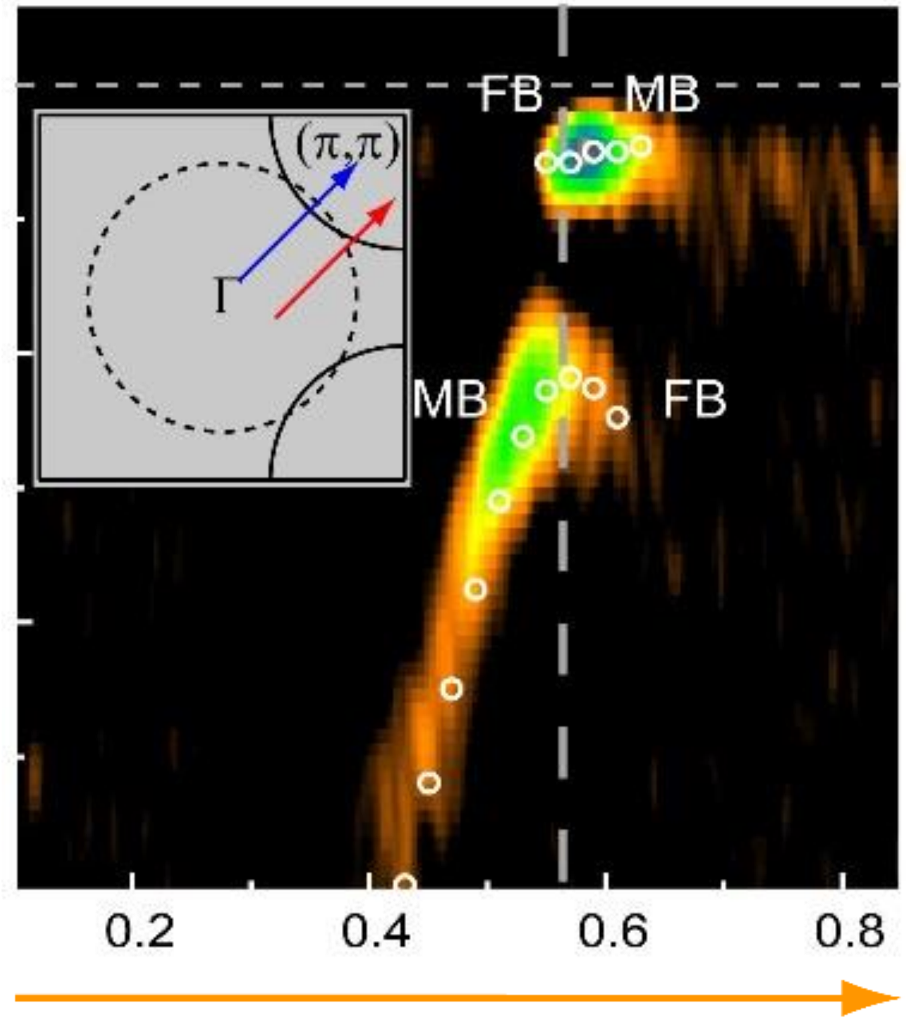
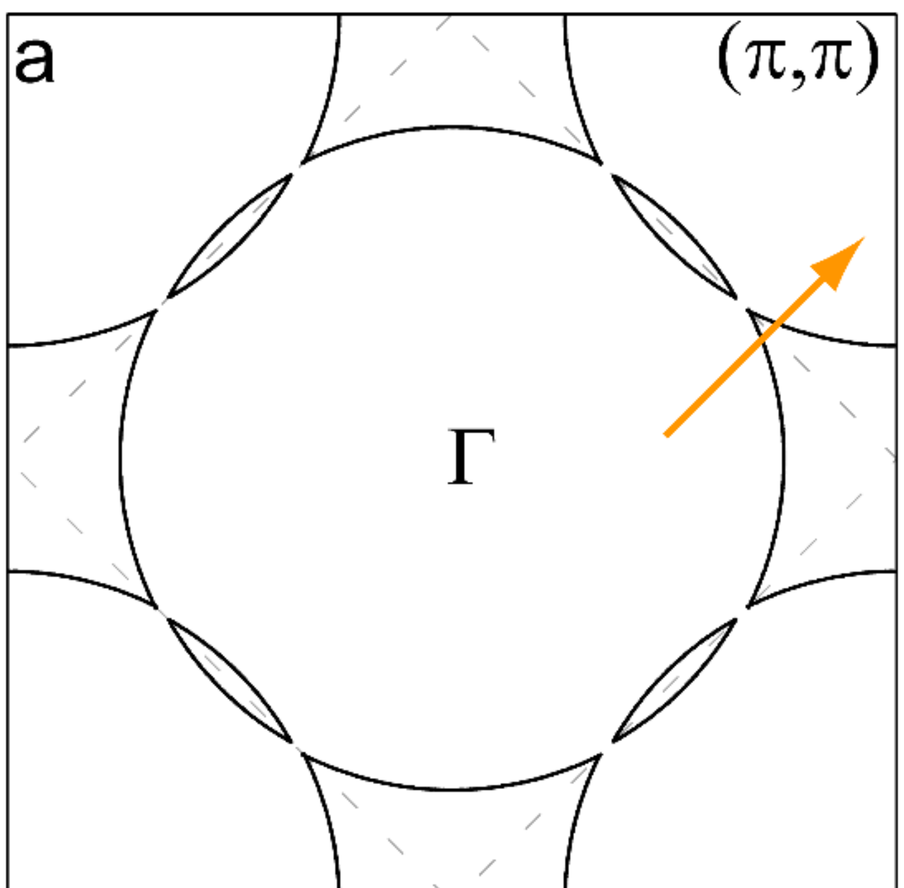
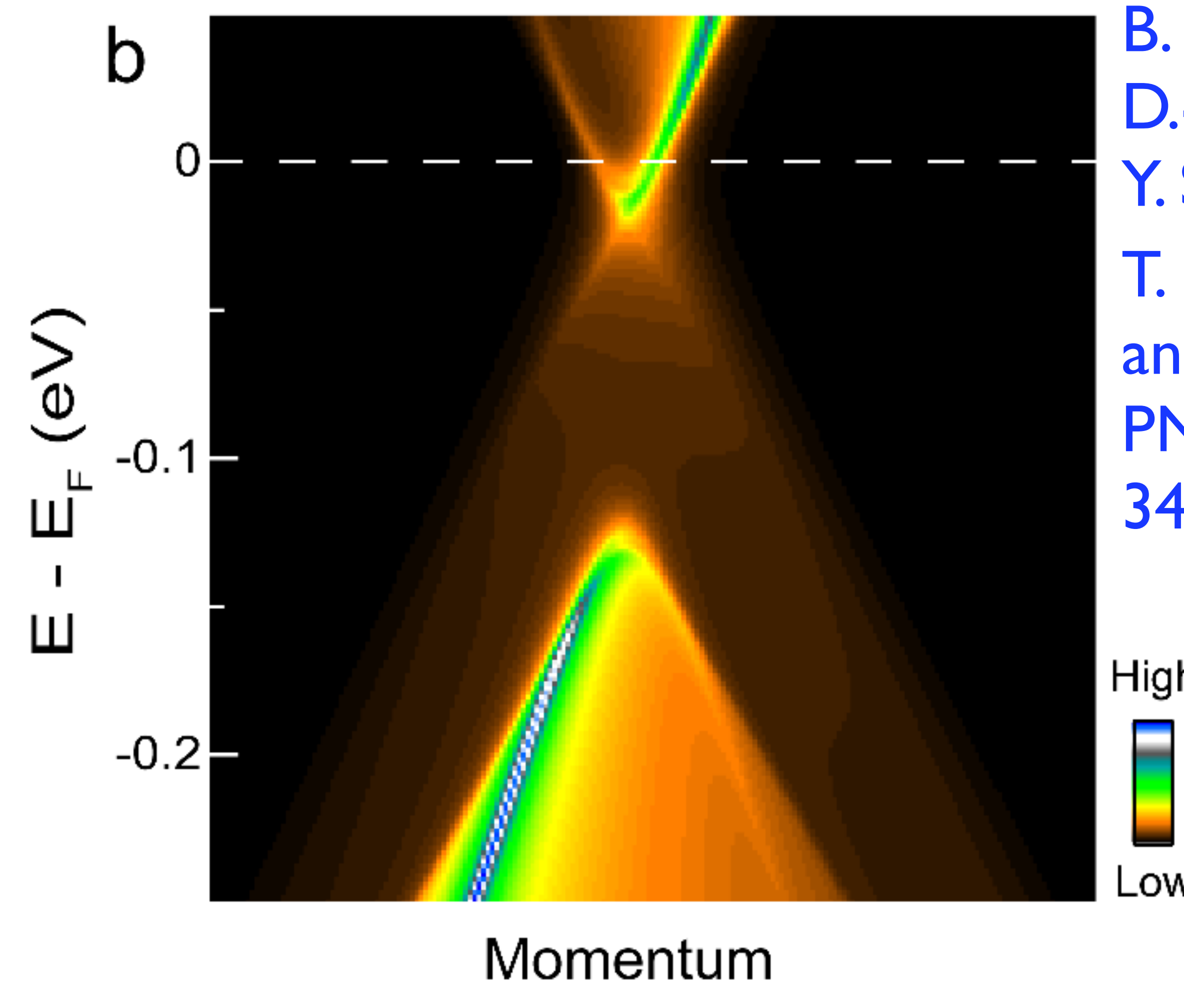
Mathias Scheurer







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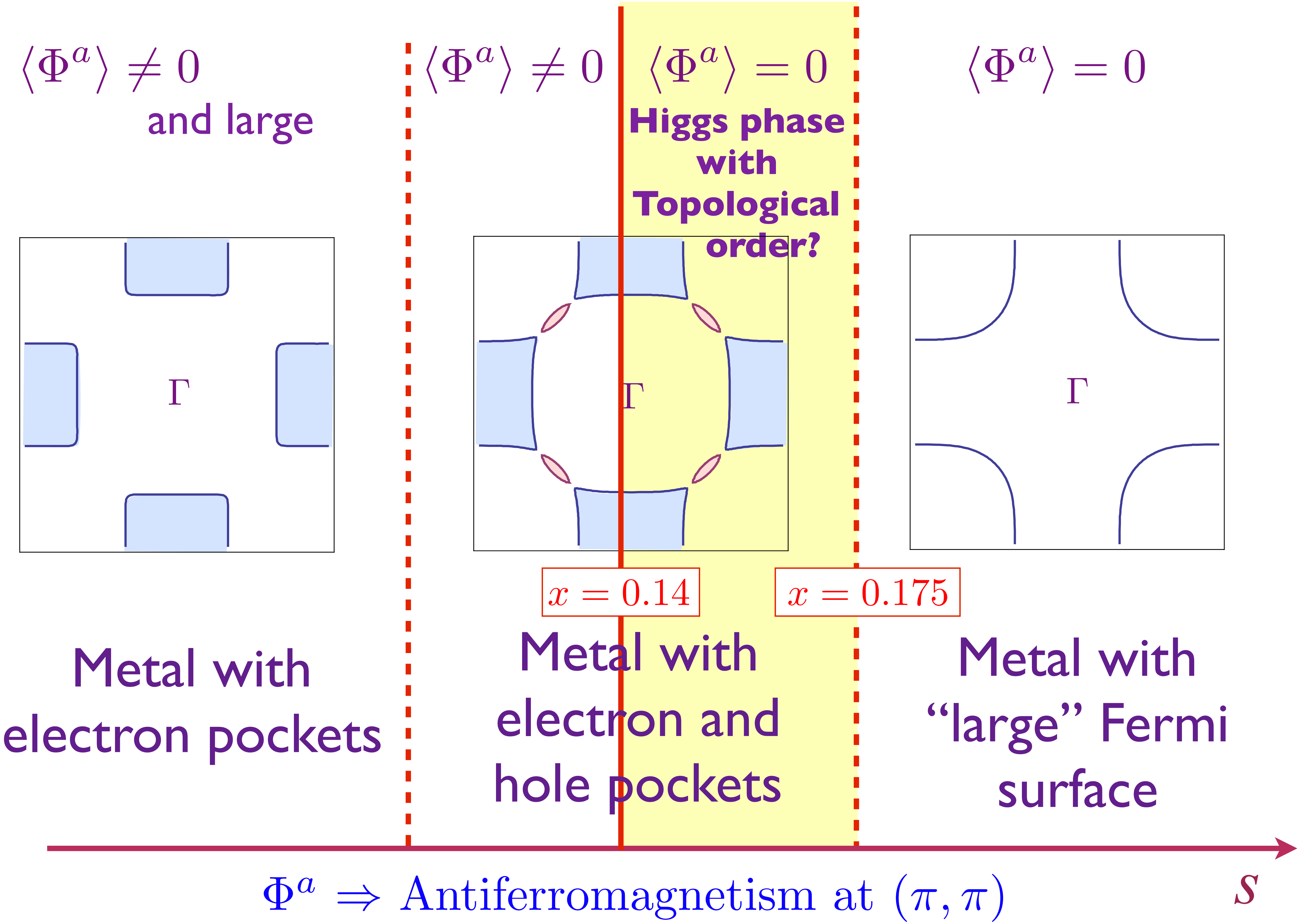
Square lattice Hubbard model with electron doping



Topological quantum matter

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- Theory of fluctuating antiferromagnetism in the electron-doped cuprates. Found a metallic state with topological order, reconstructed Fermi surfaces, and violation of the Luttinger theorem. This phase can explain recent photoemission experiments near optimal doping.