Dissipative preparation of Quantum Many-Body States

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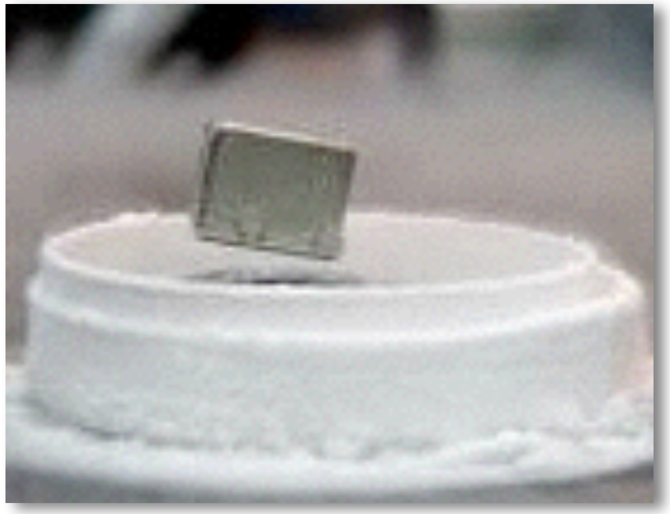
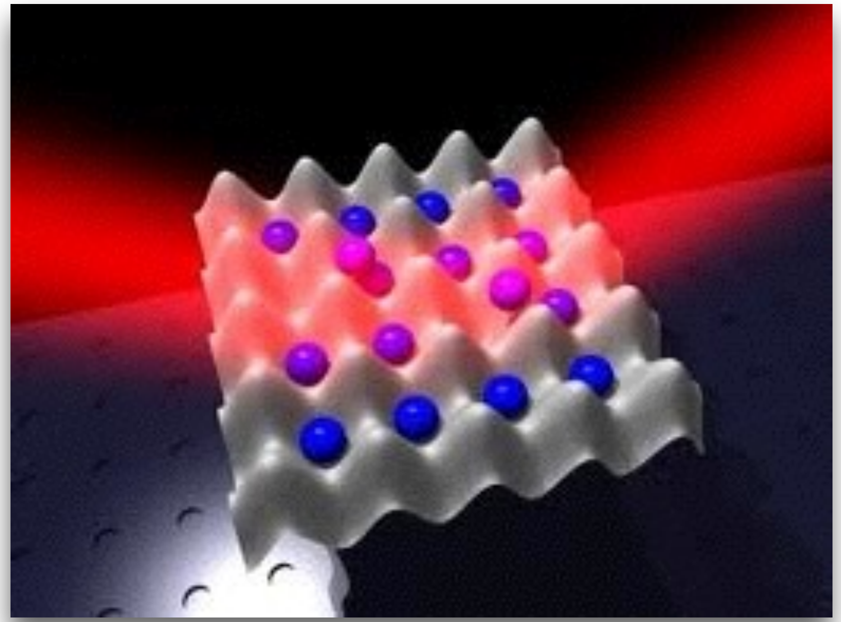
The Abdus Salam International Centre for Theoretical Physics

Condensed matter systems …
"hard to understand"



Controllable (synthetic) quantum many-body systems



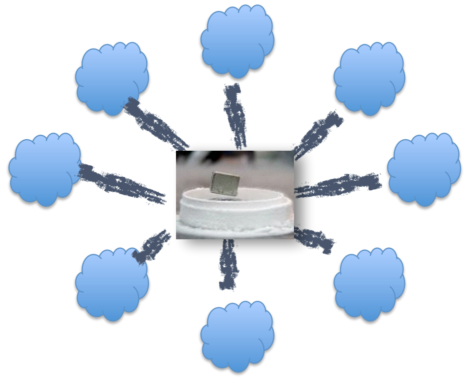
Quantum Simulators

Controllable quantum many-body systems

- Engineer specific Hamiltonians
- Highly tuneable
- *High level of coherence over large time scales*
- Good access for measurements

- Study the equilibrium properties
- Follow the (non-equilibrium) dynamics
- Preparation of many-body quantum states
- …

Dissipation in many-body systems



Hamiltonian engineering

the coupling to an external environment is detrimental

Question:

Is it possible to engineer the environment to perform quantum information protocols?

… understanding the dynamics of many-body open systems

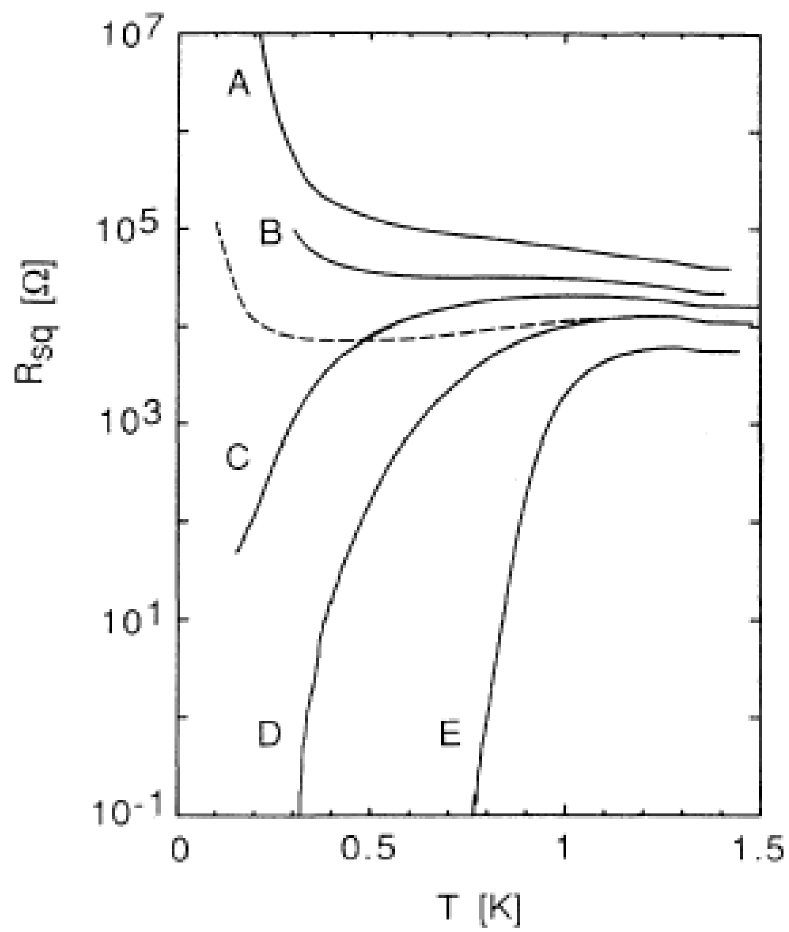
“Pre”-history

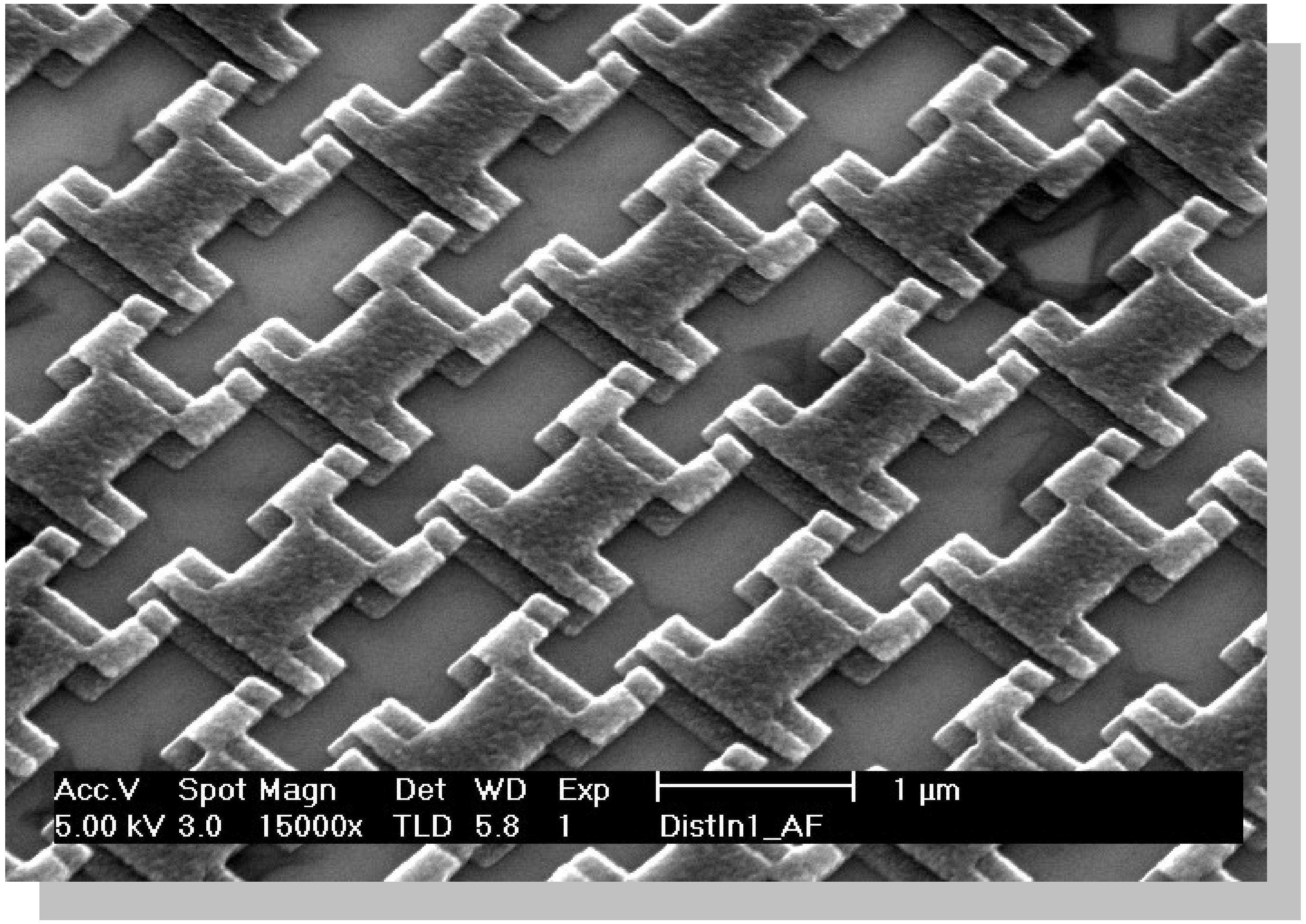
Before Quantum Information

- Effect of dissipation and macroscopic quantum dynamics (Caldeira-Leggett, Larkin-Ovchinnikov, Schmid, Ambegaokar-Eckern-Schoen, Hanggi, Weiss, Grabert, Ingold, …)

- Josephson junction arrays *(“prehistory” of quantum simulators)* (Chakravarty, Ingold, Zimanyi, Schoen, Eckern, Mooij, Kivelson, Ingold, Kampf, Eckern-Schmid, …)

Josephson junction arrays





J.E. Mooij group

VOLUME 63, NUMBER 3 PHYSICAL REVIEW LETTERS 17 JULY 1989

Charging Effects and Quantum Coherence in Regular Josephson Junction Arrays

L. J. Geerligs, M. Peters, L. E. M. de Groot,[a] A. Verbruggen,[a] and J. E. Mooij
Department of Applied Physics, Delft University of Technology, P.O. Box 5046, 2600 GA Delft, The Netherlands
(Received 17 April 1989)

Two-dimensional arrays of very-small-capacitance Josephson junctions have been studied. At low temperatures the arrays show a transition from superconducting to insulating behavior when the ratio of charging energy to Josephson-coupling energy exceeds the value 1. Insulating behavior coincides with the occurrence of a charging gap inside the BCS gap, with an S-shaped *I-V* characteristic. This so far unobserved S shape is predicted to arise from macroscopic quantum coherent effects including Bloch oscillations.

Josephson junction arrays

VOLUME 85, NUMBER 9 PHYSICAL REVIEW LETTERS 28 AUGUST 2000

Superconductor-Insulator Transition in a Two-Dimensional Array of Resistively Shunted Small Josephson Junctions

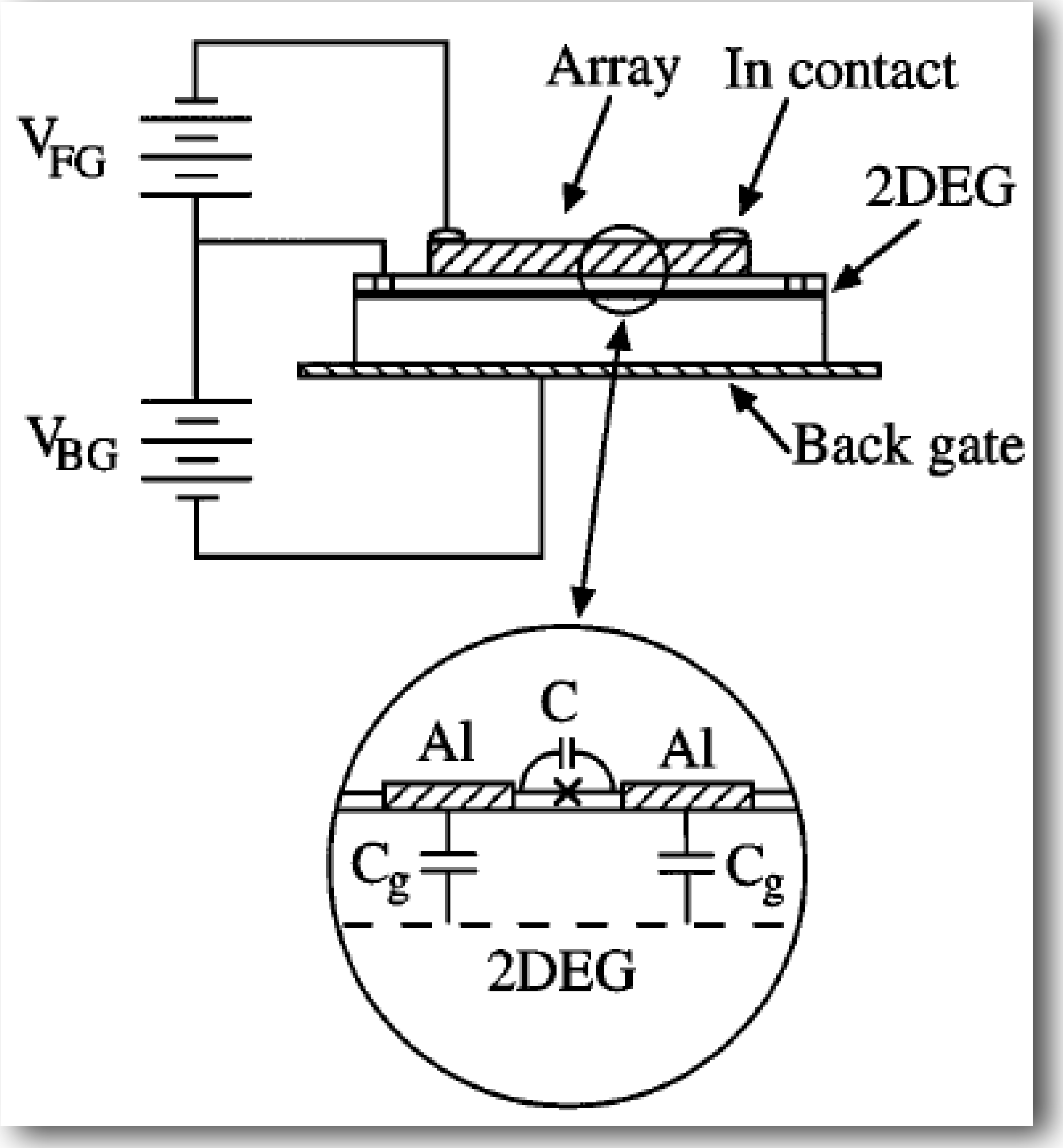
Yamaguchi Takahide,[1,2] Ryuta Yagi,[1] Akinobu Kanda,[1,2] Youiti Ootuka,[1,2] and Shun-ichi Kobayashi[3]

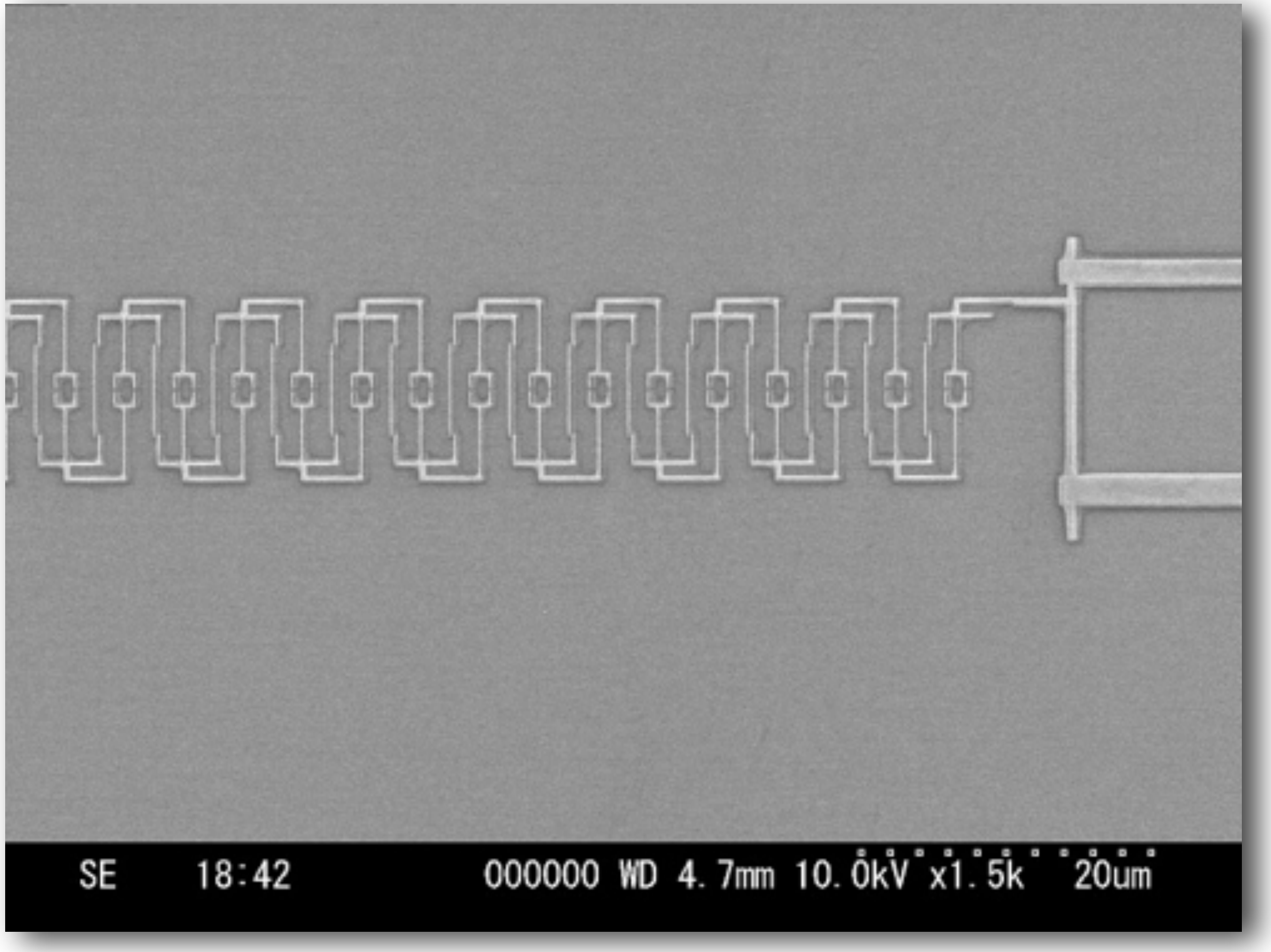
VOLUME 78, NUMBER 13 PHYSICAL REVIEW LETTERS 31 MARCH 1997

Dissipation-Driven Superconductor-Insulator Transition in a Two-Dimensional Josephson-Junction Array

A. J. Rimberg,* T. R. Ho, Ç. Kurdak, and John Clarke
Department of Physics, University of California, Berkeley, California 94720 and Materials Sciences Division, Lawrence Berkeley National Laboratory, Berkeley, California 94720

K. L. Campman and A. C. Gossard
Materials Department, University of California, Santa Barbara, California 93106
(Received 26 November 1996)



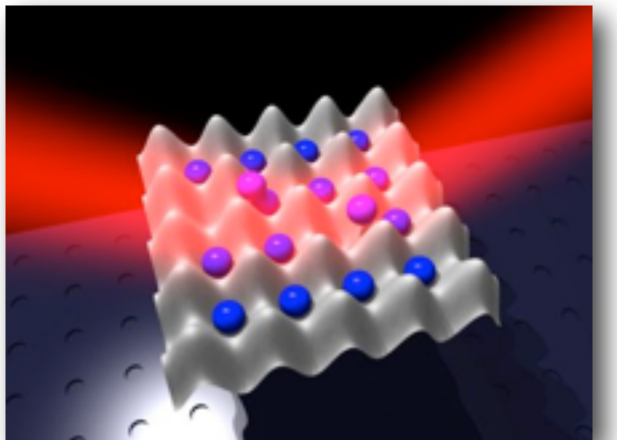


After Quantum Information

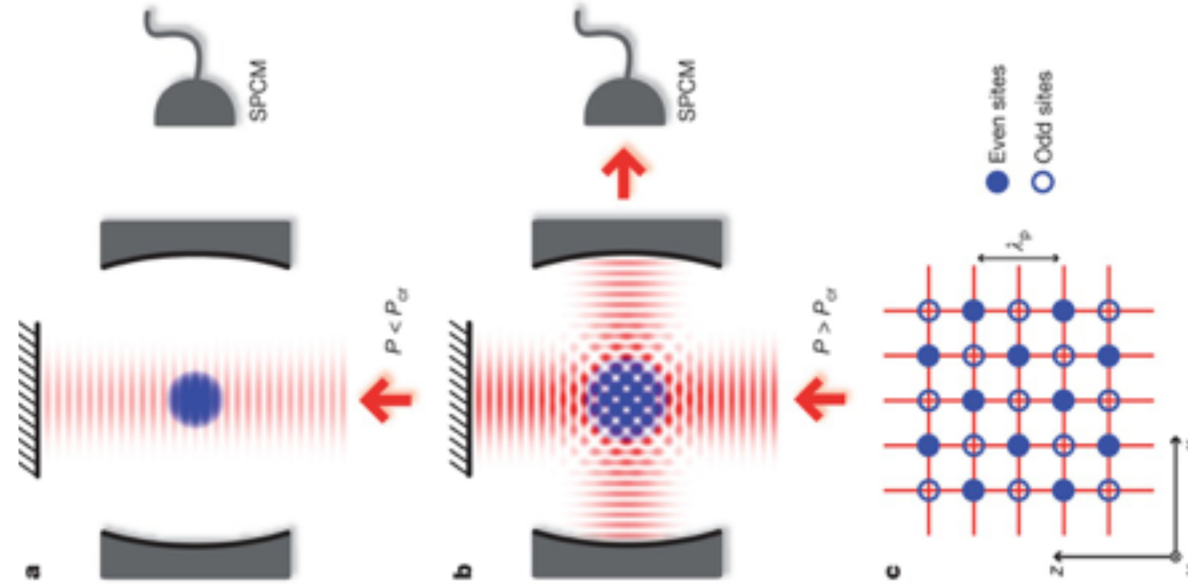
“A new twist”

- Engineered baths
- Non-equilibrium effects
- Optical lattices, cavity arrays, trapped ions, …
- **Open system quantum simulators**

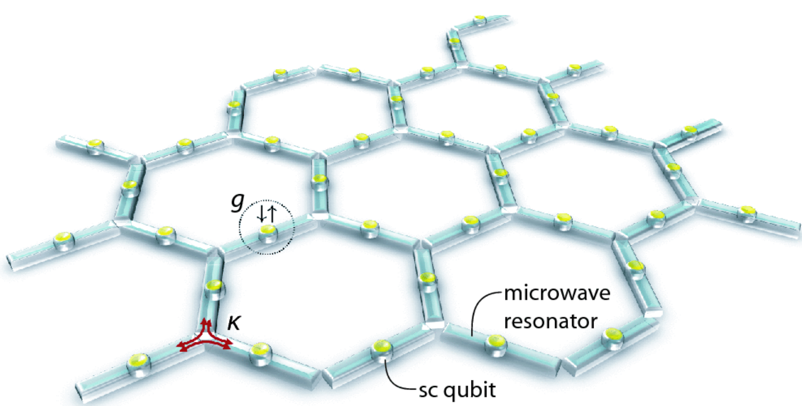
A variety of systems …



Optical lattices with engineered dissipation



BEC in cavities (Esslinger group)



Cavity arrays

…

Dissipative dynamics

$$\dot{\rho} = -i[\mathcal{H}, \rho] + \mathcal{L}[\rho]$$

Markovian bath —> Lindblad form

$$\mathcal{L}[\rho] = \sum_{\ell} \kappa_\ell \left[L_\ell \rho L_\ell^\dagger - L_\ell^\dagger L_\ell \rho - \rho L_\ell^\dagger L_\ell \right]$$

- It is the most general form that guarantees that the $\boldsymbol{\rho}$ “remains” a density matrix during the evolution
- The quantum jump operators depend on the system-bath coupling, their number is up to the Hilbert space squared
- Note: ℓ is NOT defined in real space,

- Competition within the Hamiltonian (e.g. strong local correlation and delocalisation)

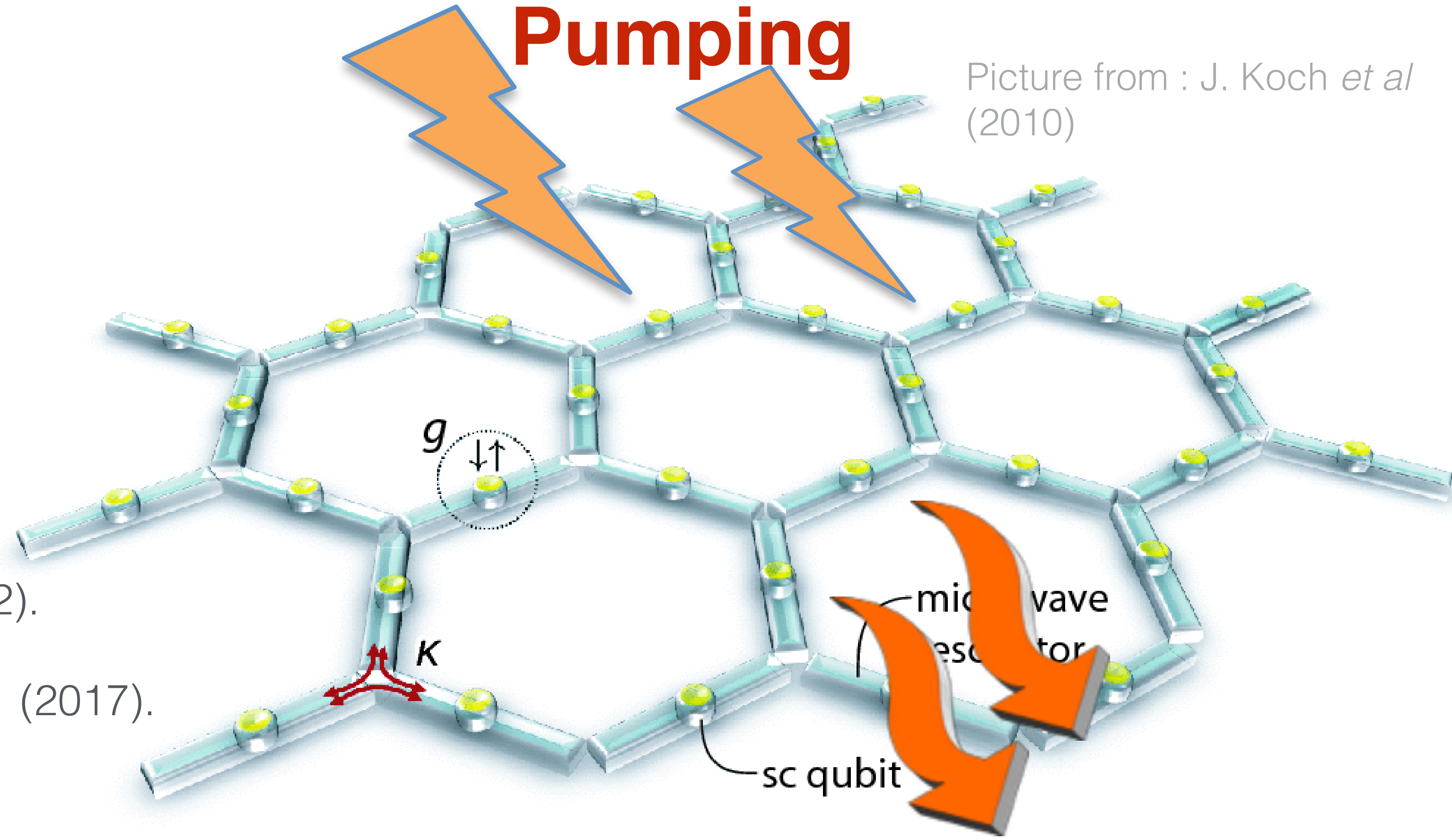
$$\mathcal{H} = \mathcal{H}_0 + g\mathcal{H}_1$$

- Competition between unitary dynamics and damping (e.g. photon leakage and external driving)

$$\mathcal{H} \longleftrightarrow \mathcal{L}$$

Coupled cavity arrays

$$\mathcal{H} = U \sum_i n_i(n_i - 1) - J \sum_{<ij>} [a_i^\dagger a_j + h.c.]$$



Picture from : J. Koch *et al* (2010)

Photon leakage

Reviews:
A. Houck, H. Tureci, and J. Koch, Nat. Phys. **8**, 292 (2012).
M. J. Hartmann, J. Opt. **18**, 104005 (2016).
C. Noh and D. G. Angelakis, Rep. Prog. Phys. **80**, 016401 (2017).

- Competition of photon leakage and external driving+Hamiltonian dynamics

coherent or incoherent

$$\mathcal{L}[\rho] = \frac{\gamma}{2} \sum_i (2 a_i \rho a_i^\dagger - \{n_i, \rho\})$$

Back to engineered baths ...

The idea is to design the form of the Lindblad operators in order to realise a desired task

$$\mathcal{L}[\rho] = \sum_\ell \kappa_\ell \left[L_\ell \rho L_\ell^\dagger - L_\ell^\dagger L_\ell \rho - \rho L_\ell^\dagger L_\ell \right]$$

S. Diehl, A. Micheli, A. Kantian, B. Kraus, H.P. Buchler, and P. Zoller Nat. Phys. (2008)
F. Verstraete, M. M. Wolf, J. I. Cirac, (2009)

The computational power of purely dissipative processes, and proven that it is equivalent to that of the quantum circuit model of quantum computation.

Any quantum circuit one can construct a master equation whose steady state is unique, encodes the outcome of the circuit

There is a route towards preparing many body states and non-equilibrium quantum phases by quantum reservoir engineering

S. Diehl, A. Micheli, A. Kantian, B. Kraus, H.P. Buchler, and P. Zoller Nat. Phys. (2008)
F. Verstraete, M. M. Wolf, J. I. Cirac, (2009)

Engineered baths

$$\dot{\rho} = \sum_{\ell} \kappa_\ell \left[L_\ell \rho L_\ell^\dagger - L_\ell^\dagger L_\ell \rho - \rho L_\ell^\dagger L_\ell \right]$$

Suppose that there is a state such that

$$L_\ell |D\rangle = 0 \quad \text{for any } \ell$$

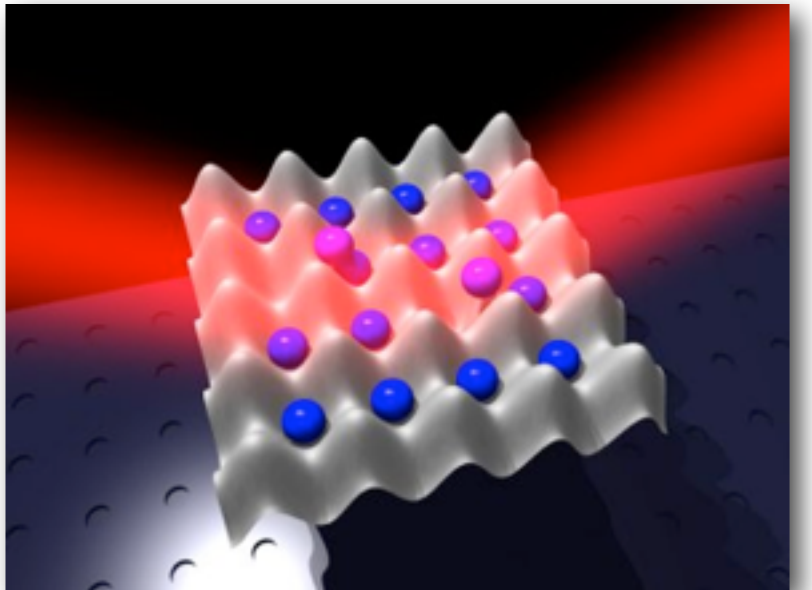
The steady state is pure and given by

$$\rho_s = |D\rangle\langle D|$$

Connection with the parent Hamiltonian

$$\mathcal{H} = \sum_{\ell} L_\ell^\dagger L_\ell$$

Engineered baths

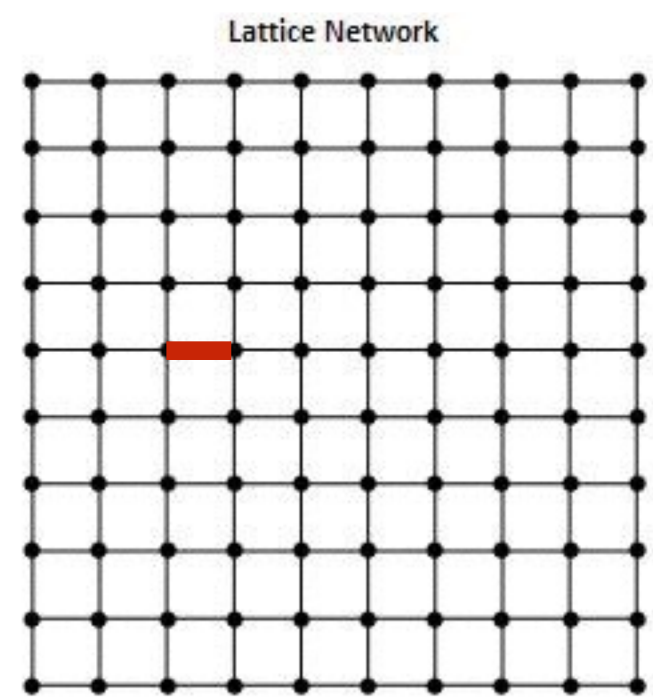


Prepare a BEC using dissipation

$$\dot{\rho} = \sum_{\ell} \kappa_\ell \left[L_\ell \rho L_\ell^\dagger - L_\ell^\dagger L_\ell \rho - \rho L_\ell^\dagger L_\ell \right]$$

Bosons on a lattice

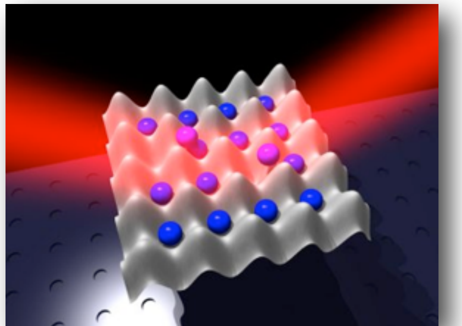
$$\underbrace{L_{<i,j>}}_{\ell} = (a_i^\dagger + a_j^\dagger)(a_i - a_j)$$



The steady state is pure and it is that of a Bose condensate

S. Diehl, A. Micheli, A. Kantian, B. Kraus, H.P. Buchler, and P. Zoller Nat. Phys. (2008)

Competition between Lindblad and Hamiltonian dynamics

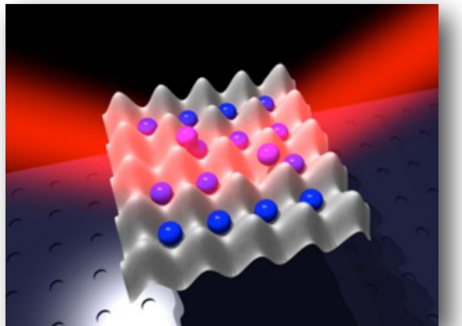


$$\dot{\rho} = -i[\mathcal{H}, \rho] + \mathcal{L}[\rho]$$

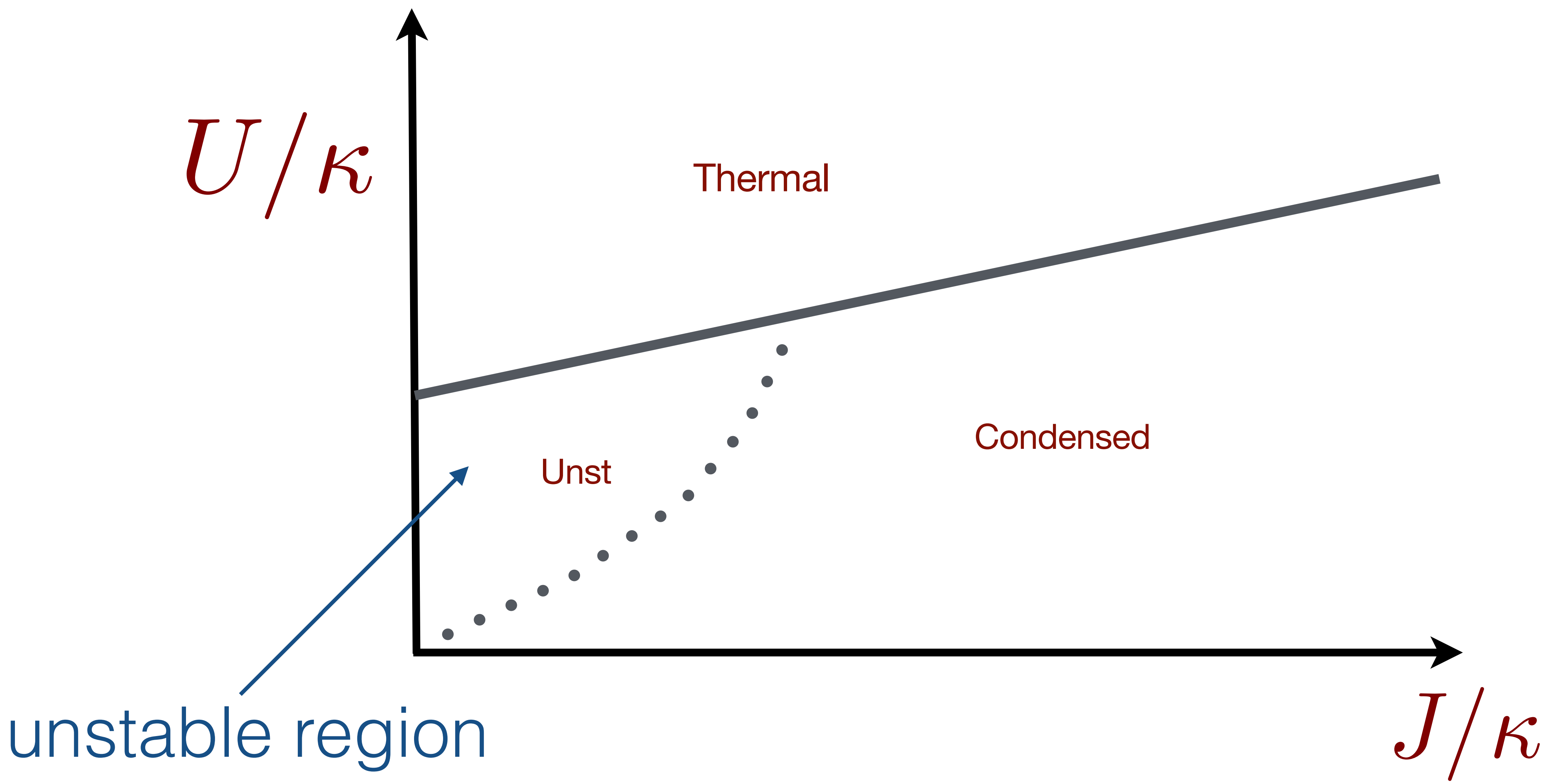
Bose-Hubbard Hamiltonian

$$\mathcal{H} = U \sum_i n_i(n_i - 1) - J \sum_{<ij>} [a_i^\dagger a_j + h.c.]$$

Competition between Lindblad and Hamiltonian dynamics



- U = 0 the steady state is a pure BEC
- U << J the steady state is “thermal” with an effective temperature ~ U



S. Diehl, A. Tomadin, A. Micheli, R. Fazio, and P. Zoller PRL (2010)

Dissipative preparation of a p-wave superconductor

S. Diehl, E. Rico, M.A. Baranov and P. Zoller, Nat. Phys. (2011).

$$\dot{\rho} = \sum_{\ell} \kappa_\ell \left[L_\ell \rho L_\ell^\dagger - L_\ell^\dagger L_\ell \rho - \rho L_\ell^\dagger L_\ell \right]$$

$$L_{<i,j>} = (a_i^\dagger + a_j^\dagger)(a_i - a_j) \qquad j = i + 1$$

Fermions on a lattice

$N \to \infty$ and $t \to \infty$

$$a_i^\dagger + a_i^\dagger + a_i - a_j$$

The dark state is the ground state of the 1D Kitaev model

Dissipative state preparation @ fixed N

- Pairing correlations

$$G^{(p)}_{i.j} = \langle a^\dagger_i a^\dagger_{i+1} a_j a_{j+1} \rangle$$

F. Iemini, D. Rossini, L. Mazza, R. Fazio, and S. Diehl, (2015)
F. Iemini, D. Rossini, R. Fazio, S. Diehl, and , L. Mazza, (2016)

“Dissipative preparation” of time-crystals

F. Iemini, A. Russomanno, J. Keeling, M. Schirò, M. Dalmonte, R.Fazio, PRL (2018)

$$\dot{\rho} = \sum_{\ell} \kappa_{\ell} \left[L_{\ell} \rho L_{\ell}^{\dagger} - L_{\ell}^{\dagger} L_{\ell} \rho - \rho L_{\ell}^{\dagger} L_{\ell} \right]$$

- Closing of the Liouvillian gap making the non-equilibrium steady state subspace degenerate in the thermodynamic limit
- Oscillating coherences appearing in the degenerate subspace
- Liouvillian gap above the degenerate subspace

Solvable model of a boundary time-crystal

$$\hat{H}_{\rm b} = \omega_0 \sum_j \hat{\sigma}_j^x \qquad \hat{S}^\alpha = \sum_j \hat{\sigma}_j^\alpha$$

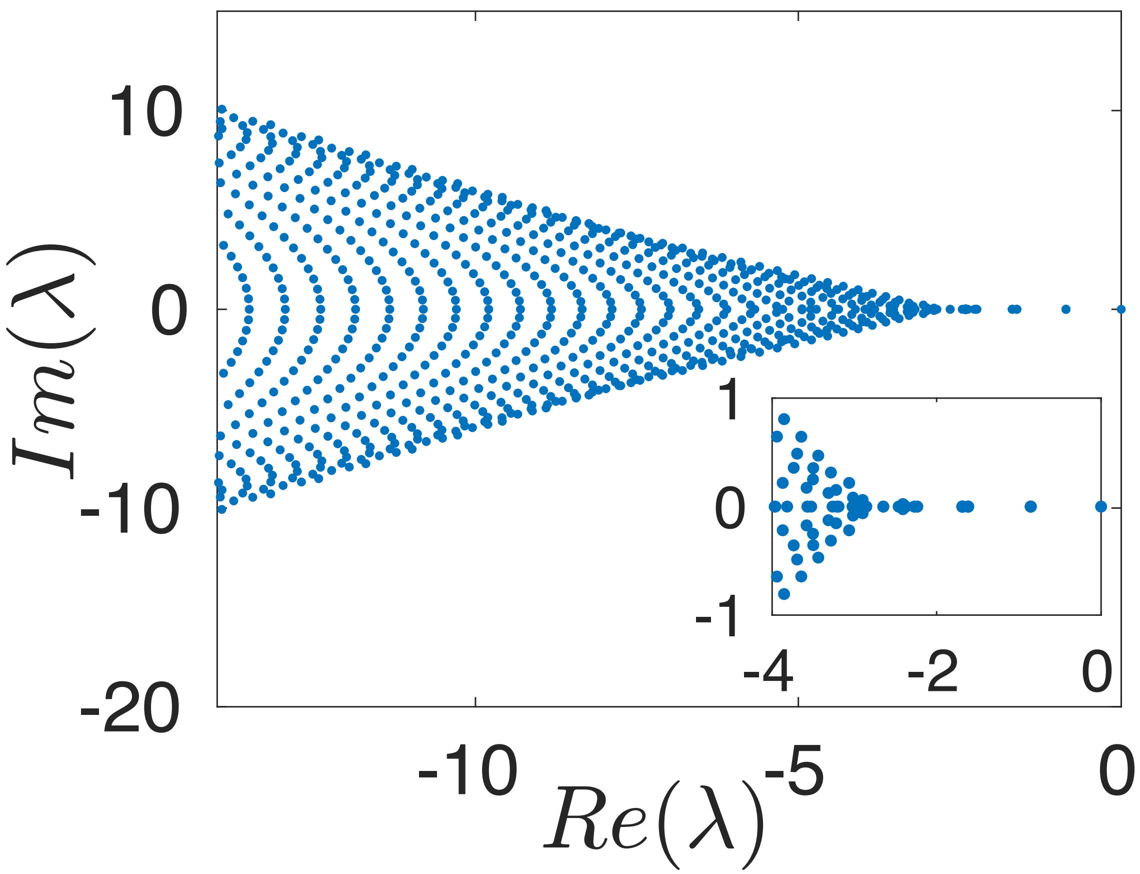
$$\frac{d}{dt}\hat{\rho}_{\rm b} = i\omega_0[\hat{\rho}_{\rm b}, \hat{S}^x] + \frac{\kappa}{S}\left(\hat{S}_-\hat{\rho}_{\rm b}\hat{S}_+ - \frac{1}{2}\{\hat{S}_+\hat{S}_-, \hat{\rho}_{\rm b}\}\right)$$

The steady state diagram of the model has two distinct phases

$\omega_0/\kappa < 1$	$\omega_0/\kappa > 1$
$\langle \hat{S}^z \rangle \neq 0$	$\langle \hat{S}^z \rangle = 0$

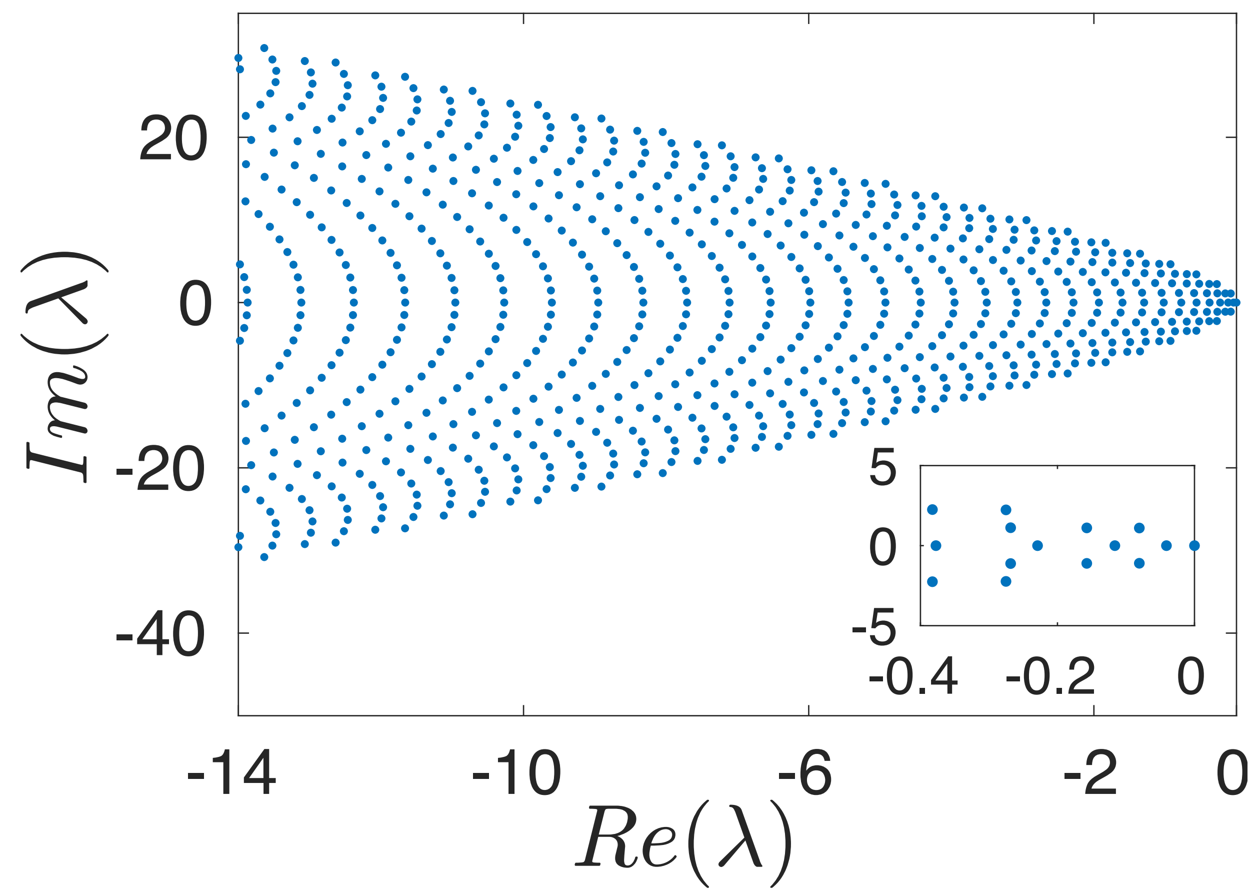
J. Hannukainen and J. Larson PRA (2017)

$$\omega_0/\kappa > 1$$

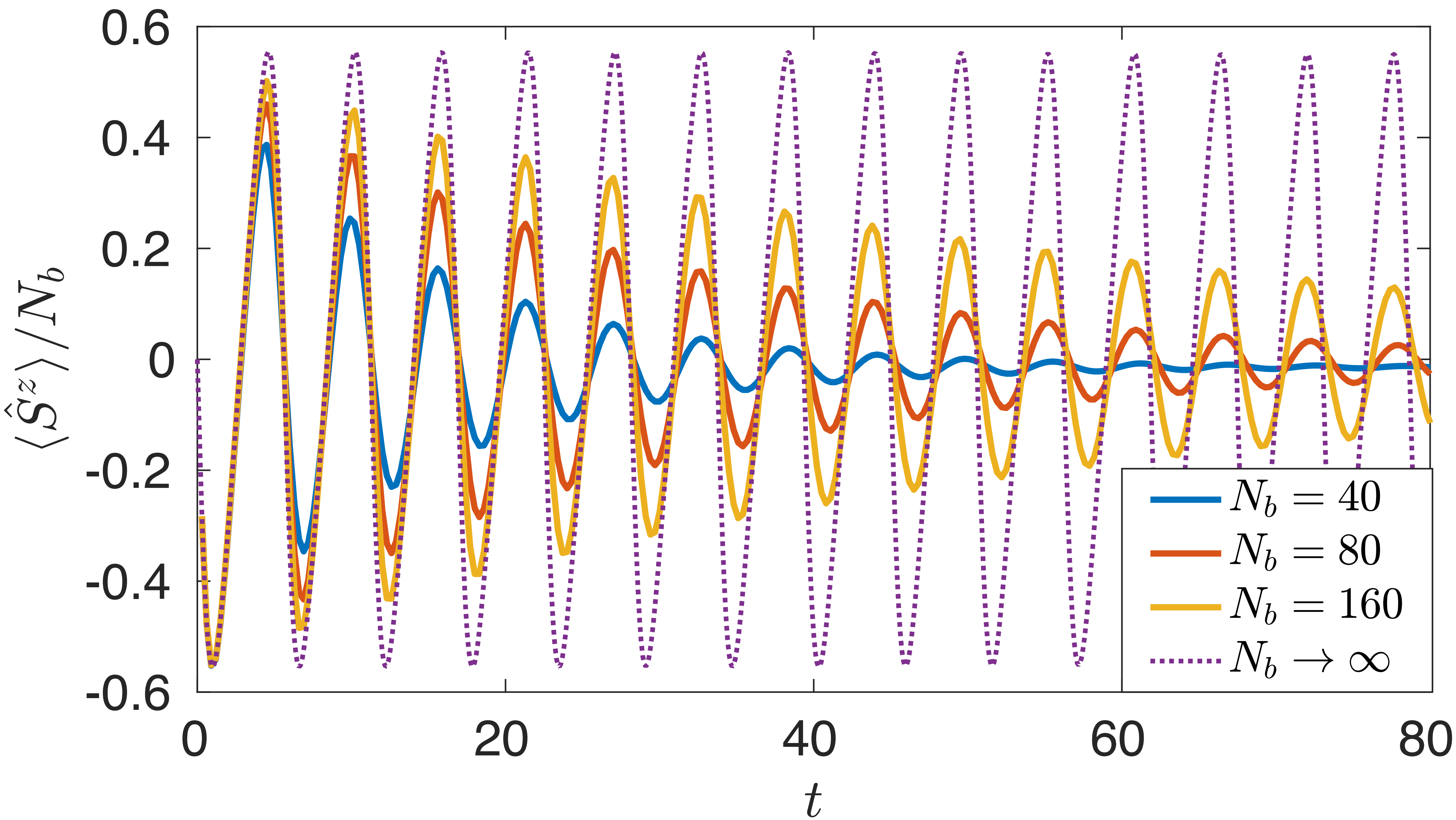


The spectrum is gapped and the low-lying eigenvalues of the Liouvillian have purely real values

$$\omega_0/\kappa < 1$$



The spectrum becomes gapless and the low-lying excited eigenvalues have a non zero imaginary part

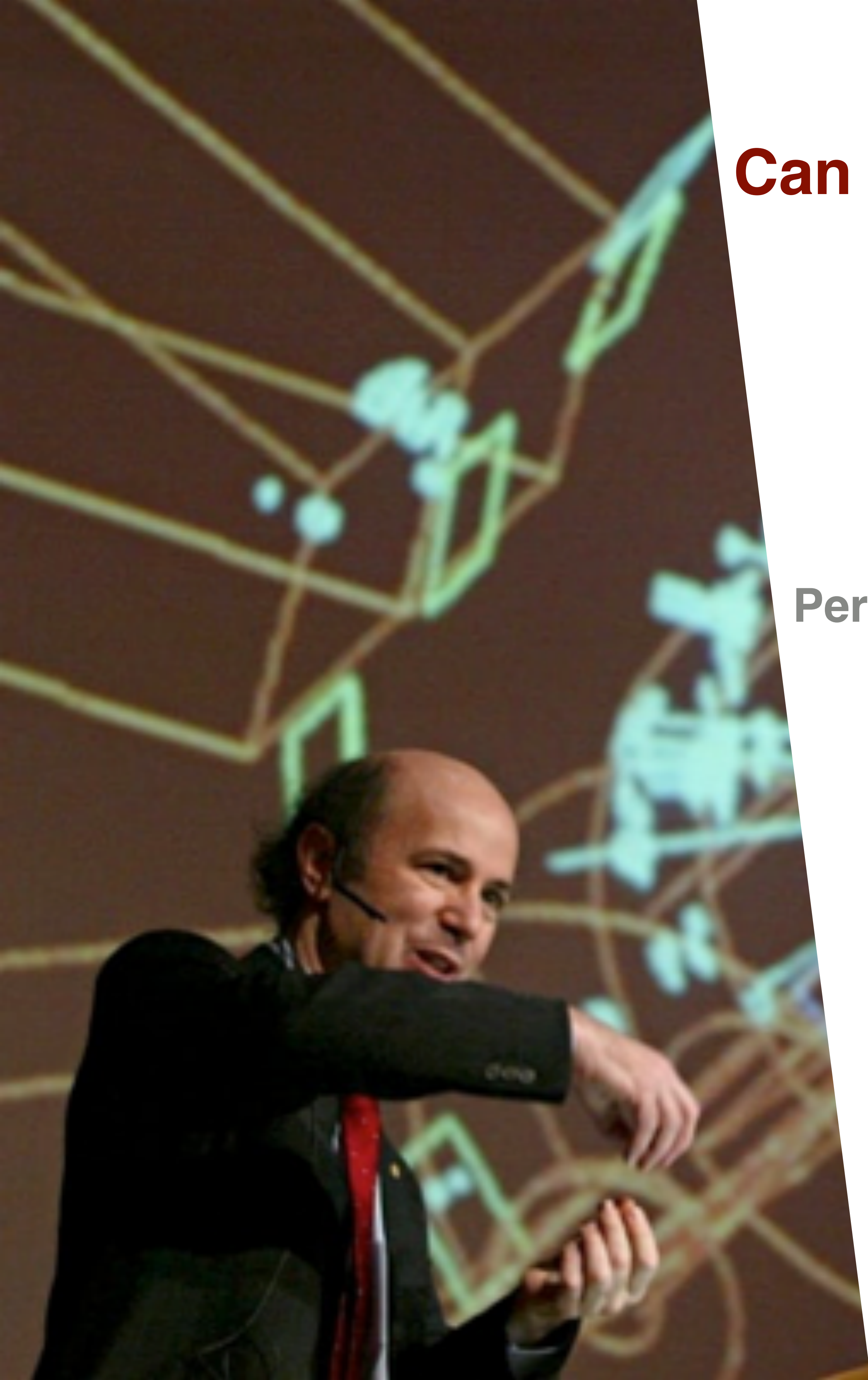


Note

Identical features have been already seen in model systems of interacting Rydberg atoms, opto-mechanical arrays, coupled cavity arrays and interacting spin-systems.

These phases were all found however in a mean-field approximation, it is not clear to which extent they will survive when fluctuations are included.

Connections to synchronisation and Time Crystals



Can time-translational invariance be spontaneously broken?

Wilczek 2013

Periodic motion of a quantum many body system

Time-crystal

Definition of a time crystal

Time-dependent Hamiltonian $\mathcal{H}$

$$< O >= g(t)$$

Bad definition:

Even Rabi oscillations would fit into the category of time-crystals

Time crystals

Definition TTSB: $\phi(\vec{x}, t)$ local order parameter

$$\lim_{V \to \infty} \langle \phi(\vec{x}, t) \phi(\vec{x'}, t') \rangle \underset{|\vec{x} - \vec{x'}| \to \infty}{\longrightarrow} f(t - t')$$

No-go theorem:(*) systems in the ground state or in thermal equilibrium cannot manifest any time-crystalline behaviour

Watanabe & Oshikawa 2015

(*) with sufficiently short-interactions

Floquet time crystals

(TTSB in periodically driven systems)

Theory
D. V. Else, B. Bauer, and C. Nayak, Phys. Rev. Lett. **117**, 090402 (2016).
V. Khemani, A. Lazarides, R. Moessner, and S. Sondhi, Phys. Rev. Lett. **116**, 250401 (2016).

Experiments
J. Zhang *et al*, Nature **543**, 217 (2017)
S. Choi *et al*, Nature **543**, 221 (2017).

$$\mathcal{H}(t+T) = \mathcal{H}(t)$$

Hamiltonian **Period T** → **Spontaneous breaking** → Observables **Period nT**

$$f(t) = \lim_{N\to\infty} \langle\psi|\hat{O}(t)|\psi\rangle$$

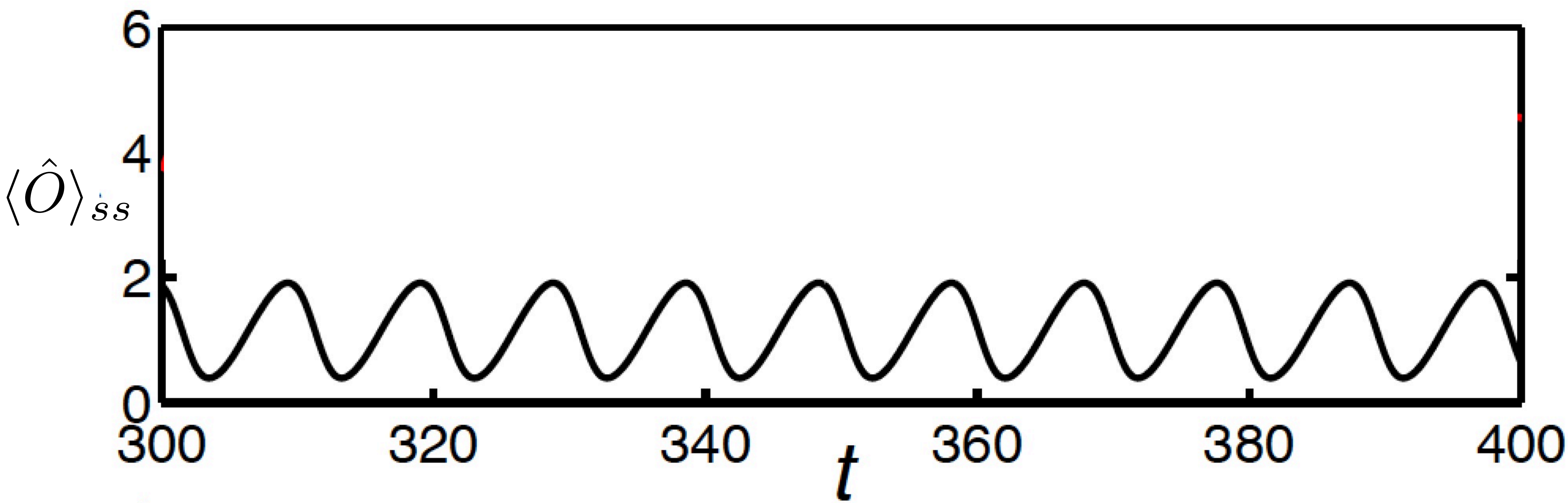
$$f(t+\tau_B) = f(t) \qquad \tau_B = nT$$

Quantum synchronisation & steady state limit cycles

Lee, Haffner, and Cross (2011)
M. Ludwig and F. Marquardt,(2013)
Jin, *et al* (2013)
Schiro', *et al* (2016)
Chan, Lee, and Gopalakrishnan (2015)
…

$\hat{O}$ macroscopic order parameter

$$\langle \hat{O} \rangle_{ss} = \mathrm{Tr} \rho_{ss} \hat{O} = f(t)$$



Conclusions

- Dissipation is not always “a problem”
- Many-Body state preparation
- Possibility of “exotic” phases