Majorana bound states: on scales and symmetries

Anton Akhmerov with Tom Laeven, André Melo, Bas Nijholt, Sebastian Rubbert, and Michael Wimmer

PRB **93**, 235434 (2016) & arXiv:1903.06168 & arXiv:1905.02725 (+code).

ICTP Summer School "Advances in Condensed Matter Physics", 10 May 2019

TU Delft — Delft University of Technology

Quantum Tinkerer

Plan

I will explain

- why Majoranas are hard to make;
- why symmetries are *really* important;
- what cool tricks one can play to make Majoranas.

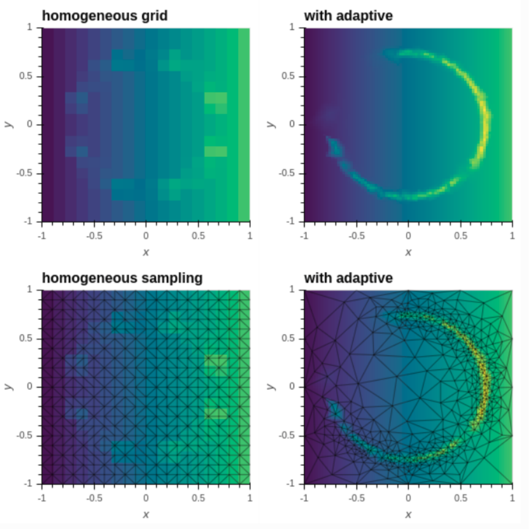
I won't tell anything about

- how to detect Majoranas and distinguish them from fake Majoranas;
- how we can use Majoranas for quantum computing;
- what happens in ongoing experiments.

But happy to chat afterwards!

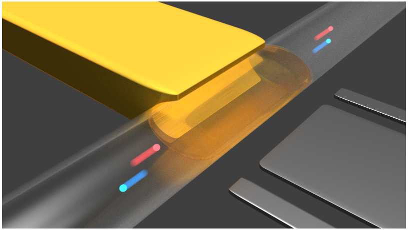
Adaptive

Our simulations use the `adaptive` library for samping.



Part I: 3D

Majorana bound states: Basic properties



Majoranas in a minimal model

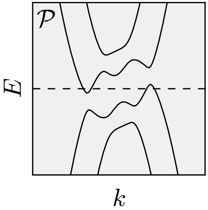
$$H = \tau_z \left(p^2/2m + \alpha p \sigma_y\right) + \Delta\tau_x + E_{\mathrm{Z}}\sigma_z$$

appear when

$$E_{\mathrm{Z}}^2 > \Delta^2 + \mu^2$$

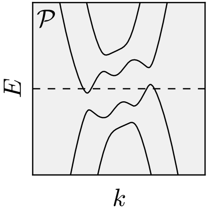
Q: Do we miss anything?

Symmetries and protection



- Particle-hole symmetry $E(k) = -E(-k)$ not good enough to guarantee the gap.

Symmetries and protection



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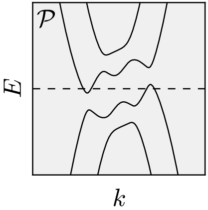

- Particle-hole symmetry $E(k) = -E(-k)$ not good enough to guarantee the gap.
- Tilting is a consequence of broken time reversal $\Rightarrow$ we should be worried.
- Chiral ("BDI") symmetry gives $\mathcal{C}H(k)\mathcal{C} = -H(k)$ and holds when $l_{SO} < d$.

Symmetries: what is available?

A less minimal model:

$$H_{BdG} = \left(\frac{\mathbf{p}^2}{2m^*} - \mu\right)\tau_z + C(\mathbf{E}\cdot\boldsymbol{\sigma}\times\mathbf{p})\tau_z + \frac{1}{2}g\mu_B\mathbf{B}\cdot\boldsymbol{\sigma} + \Delta\tau_x.$$

$$\mathbf{p} = i\nabla + e\mathbf{A}\tau_z$$

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- B_y breaks $\mathcal{C}$ (known), but so does $\mathbf{A}$!

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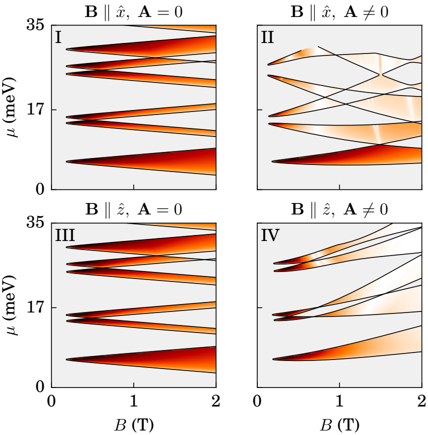
- B_y breaks $\mathcal{C}$ (known), but so does $\mathbf{A}$!
- Let's search for all possible symmetries
- There can be two more:
  - If $B_y = 0$: chiral + reflection $\mathcal{C}'H\mathcal{C}' = -H$, $\mathcal{C}' = \sigma_y\tau_y\delta(y+y')$
  - If $B_y = B_z = 0$: x-reflection $\sigma_x\tau_0 H(k)\sigma_x\tau_0 = H(-k)$.

Phase diagrams

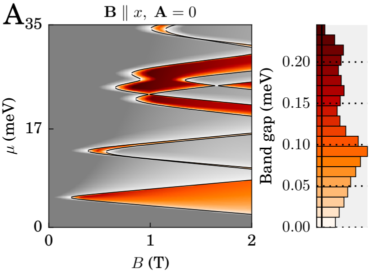
$\Delta = \text{const}$



Orbital effects are stronger than Zeeman

Phase diagrams

Proximity superconductivity:



The phase diagram is very sensitive to geometry and parameters.

Conclusions I

Orbital field:

- Point $\boldsymbol{B}$ along the wire :-/
- Keep chemical potential low :-/
- Hard to make a T-junction :-(

Part II: Supercurrents

Can we make Josephson junctions better?

Consider:

1. Josephson junctions lower the critical field (Ady's talk)
2. Phase differences break time reversal symmetry
3. We need more than 1 phase difference to close the gap (van Heck, Mi, AA)

Q: can we make Majoranas in Josephson junctions without magnetic field?

Can we make Josephson junctions better?

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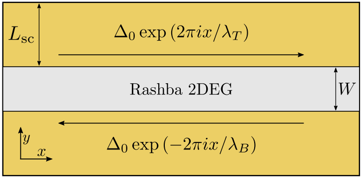
Q: can we make Majoranas in Josephson junctions without magnetic field?
A: Yes, but it we will have to work for it!

Step 1

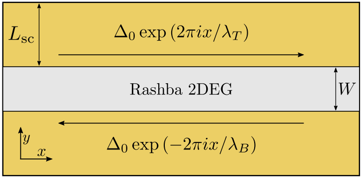
Minimal system, 2 supercurrents

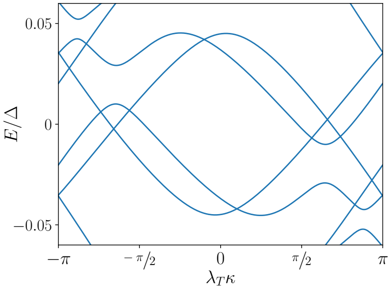


Step 1

Minimal system, 2 supercurrents

1. Try $\lambda_T = 2 \times \lambda_B$





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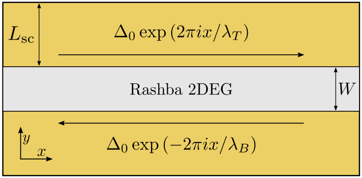

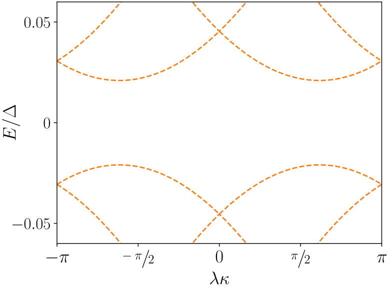

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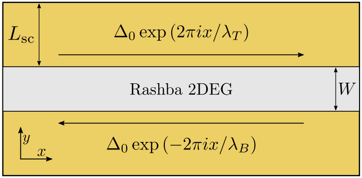
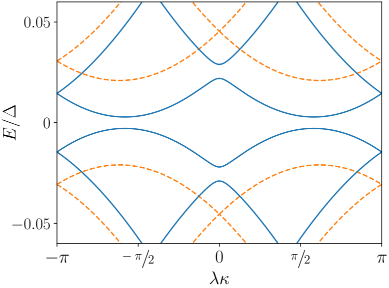
3. $W \sim l_{SO}$, ✓ spin splitting





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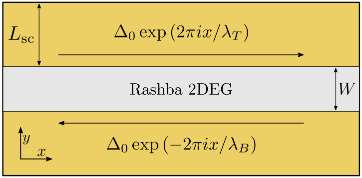
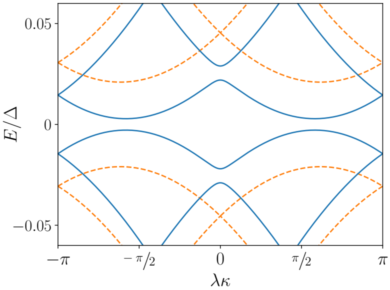
   + mysterious symmetry

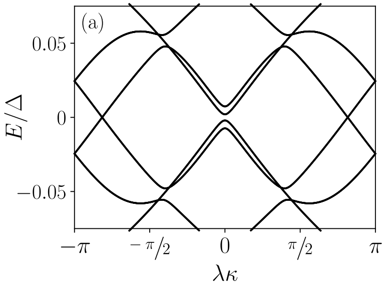




Step 2

Tuning to the topological regime

1. Tune $\mu \Rightarrow$ No gap!?



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Tuning to the topological regime

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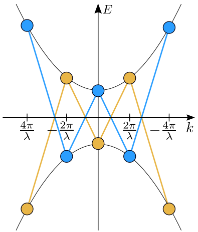
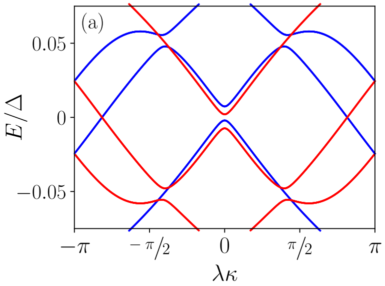
2. Charge-momentum conservation law $\mathcal{O} = (-1)^n \tau_z$





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Tuning to the topological regime

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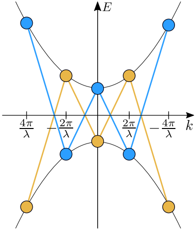
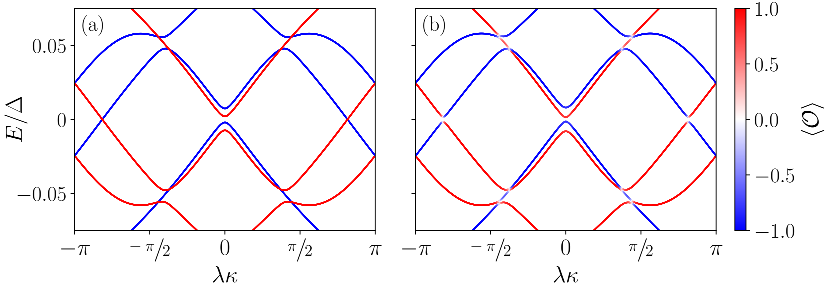
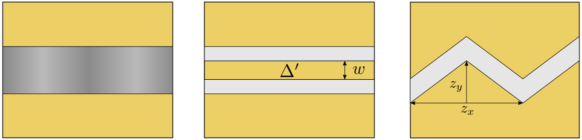
3. Broken by $V \sim \cos(2\pi x/\lambda_V)$





Step 3: Robustness check

Trying periodic potential + 2 more:



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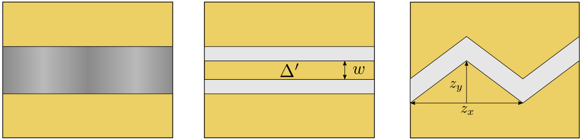
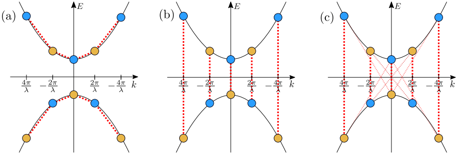


Symmetry breaking



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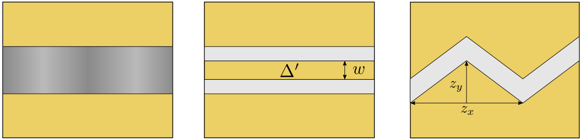

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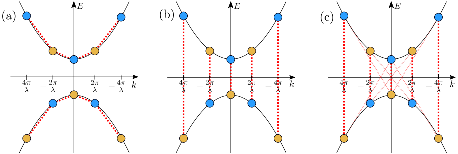
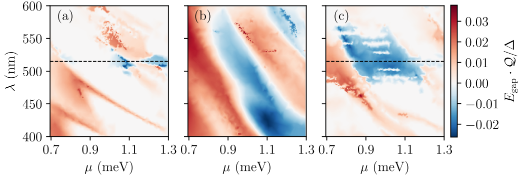


Phase diagrams



Conclusions part II

1. We created Majoranas without magnetic field :-)
2. We had to discover 4 extra symmetries 8-o
3. The topological gap is still small, needs more work :-(

Part III: Zigzag

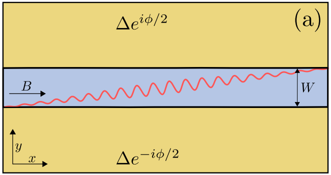
The problem of Josephson junction Majoranas

Long trajectories ⇒ tiny Δ

Workarounds:

- Low density, makes everything more disordered :-(
- Disorder, need just the right amount :-(

Q: Can we remove long trajectories by design? (Tom Laeven)



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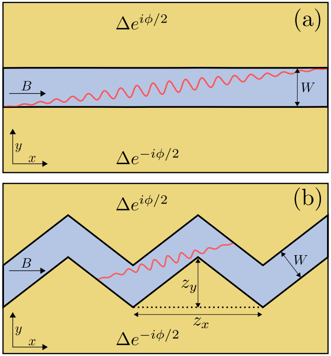

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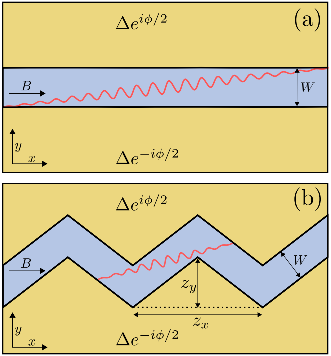
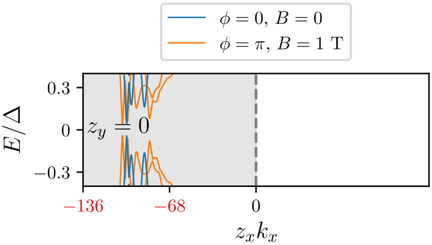
Q: Can we remove long trajectories by design? (Tom Laeven)
A: Nah, you are just going to break everything (me)



Band structures



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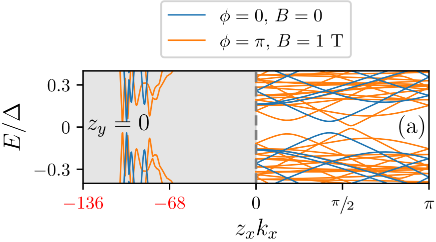

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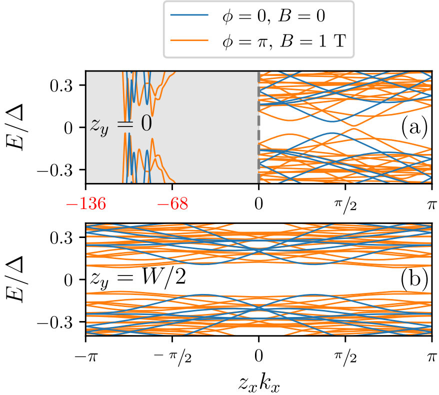

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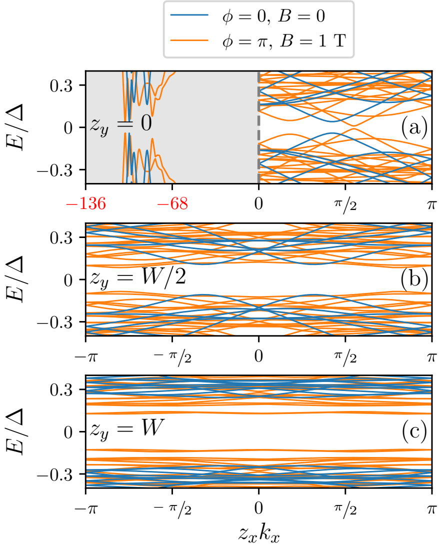

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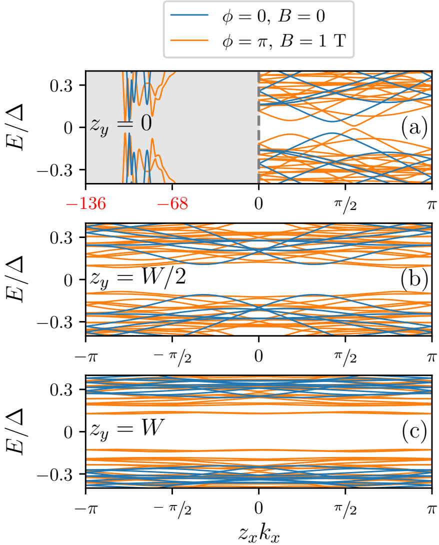
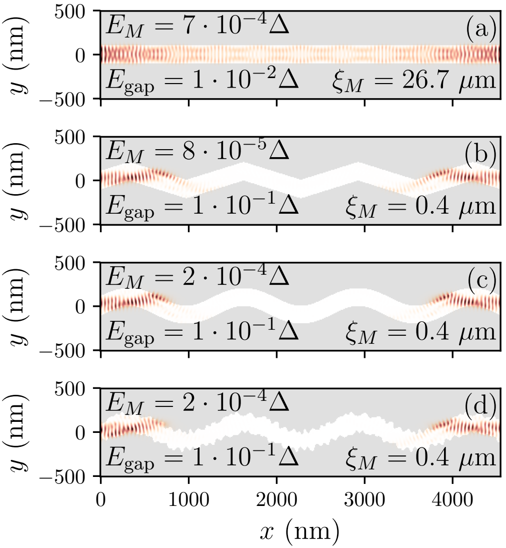


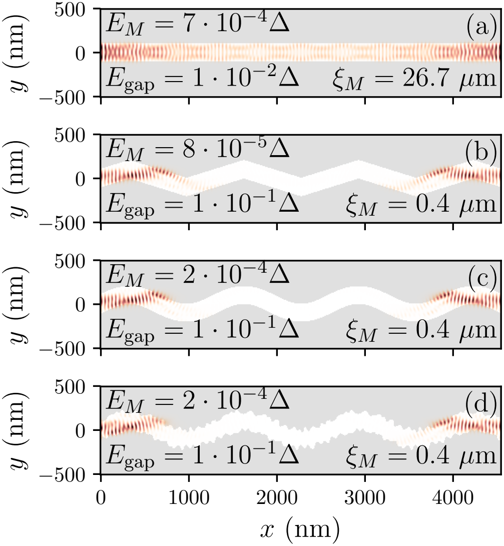
OK, but does it have Majoranas?

Majoranas



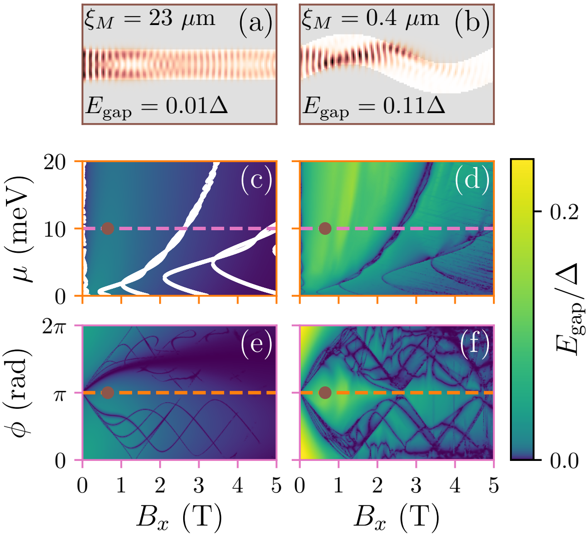
$E_M = 7 \cdot 10^{-4}\Delta$
(a)
$E_{\text{gap}} = 1 \cdot 10^{-2}\Delta$
$\xi_M = 26.7\ \mu\text{m}$
$E_M = 8 \cdot 10^{-5}\Delta$
(b)
$E_{\text{gap}} = 1 \cdot 10^{-1}\Delta$
$\xi_M = 0.4\ \mu\text{m}$
$E_M = 2 \cdot 10^{-4}\Delta$
(c)
$E_{\text{gap}} = 1 \cdot 10^{-1}\Delta$
$\xi_M = 0.4\ \mu\text{m}$
$E_M = 2 \cdot 10^{-4}\Delta$
(d)
$E_{\text{gap}} = 1 \cdot 10^{-1}\Delta$
$\xi_M = 0.4\ \mu\text{m}$
y (nm)
500
0
−500
0
1000
2000
3000
4000
x (nm)

Majoranas

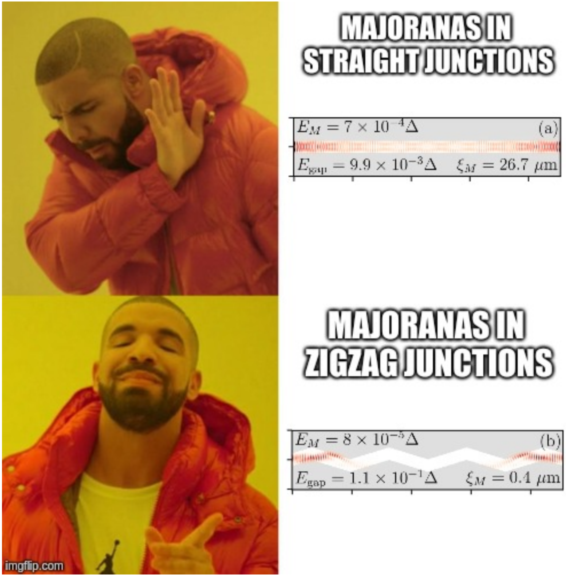


Oh wow! But maybe you are very lucky?

Topological phase diagram

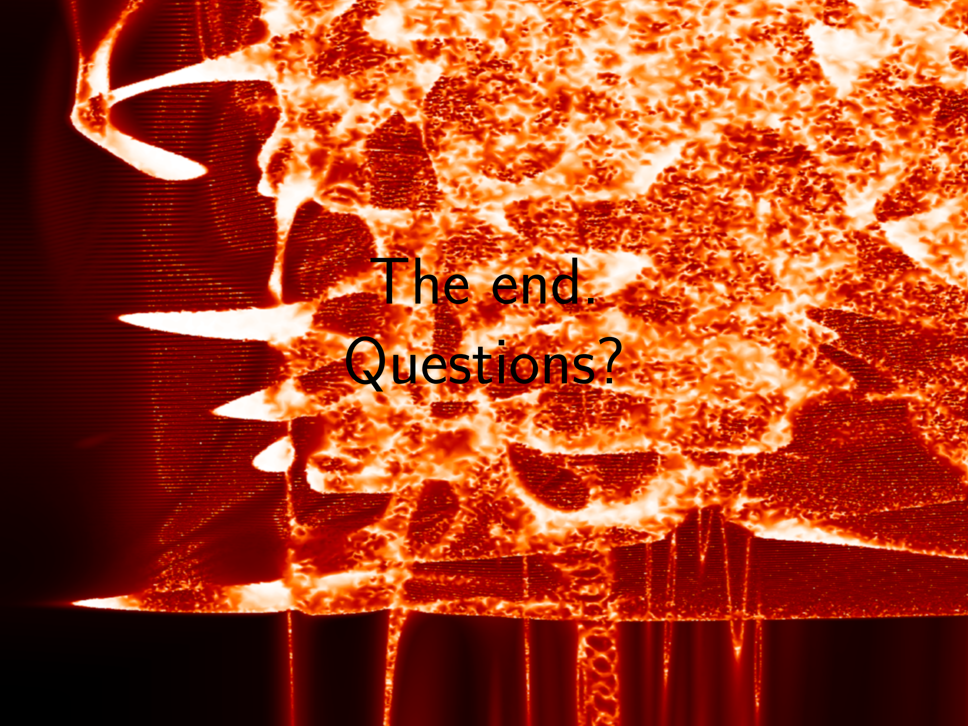


Conclusions part III



Conclusions

- Majoranas require control over several competing and complex phenomena.
- Their complexity also offers unexpected approaches.
- Symmetry considerations, scaling analysis, and simulations work combine to a powerful tool.


The end.
Questions?