Spin filters: writing quantum information on flying qubits

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Outline

- Quantum computers: qubits
- Spin-orbit interaction (SOI) and spin filters
- Time reversal symmetry no filtering with 2 leads
- Interferometers with Aharonov-Bohm flux or with leakage
- Interferometers with 3 leads
- Datta-Das Spin Field Effect Transistor
- Helical molecular junctions

Quantum computers

Conventional computers: information in bits,

0 or 1, +1 or -1, 1 or 🖡

Quantum computers: information in Qubits,

Electron described by **spinor**:

$$\psi = \cos \alpha \begin{pmatrix} 1 \\ 0 \end{pmatrix} + e^{i\gamma} \sin \alpha \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Complex numbers

Spinor is an eigenvector of $\, {f n} \, \cdot \, {m \sigma} \,$, the spin component along $\, {f n} \,$



superposition







cat lives with probability $|a|^2$ and dies with probability $|b|^2$

$$\psi = a | 0 \rangle + b | 1 \rangle$$
$$\psi = a \begin{pmatrix} 1 \\ 0 \end{pmatrix} + b \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$



Electronic spin



$$|\psi\rangle = \alpha |\uparrow\rangle + \beta |\downarrow\rangle$$

Static qubits:





Here we discuss **mobile** (or **flying**) qubits, in **mesoscopic semiconductor** devices



Flying qubits

'Writing' on flying spinor (=Qubit): Spin filtering

Work with **mobile** electrons,

Generate fully spin-polarized current out of an unpolarized source



The Aharonov-Bohm (AB) Effect

Classical Physics, e.g. Lorentz force

$$\mathbf{m}\left(\frac{\mathbf{d}^{2}\vec{\mathbf{r}}}{\mathbf{dt}^{2}}\right) = -\mathbf{e}\left[\vec{\mathbf{E}} + \frac{\vec{\mathbf{v}}}{\mathbf{c}} \times \vec{\mathbf{B}}\right]$$

Quantum Physics, Schrödinger equation

$$(H_{o} + V)\Psi = \frac{i\hbar\partial\Psi}{\partial t}$$

$$V = -eED \text{ and } H_{o} = \frac{p^{2}}{2m} \text{ , where } \vec{p} = m\vec{v} + \frac{e\vec{A}}{c}$$

with E electric field, D electrode's separation

Aharonov and Bohm (AB), Phys.Rev. 115, 485 (1959)

Phase Shift
$$\Delta \phi = \frac{1}{\hbar} \int L dt = \frac{1}{\hbar} \int \left(m \vec{v} + \frac{e \vec{A}}{c} \right) d\vec{s} - \frac{1}{\hbar} \int eED dt$$





Aharonov-Bohm interferometer





Rashba Spin-orbit interactions

Dirac:

$$\hat{H}_{SO} = \frac{\hbar}{(2M_0c)^2} \nabla V(\mathbf{r}) (\hat{\boldsymbol{\sigma}} \times \hat{\mathbf{p}}).$$
Rashba: 2DEG, confined to a plane by an asymmetric potential along z:

$$\mathcal{H}_{R} = \frac{\hbar k_R}{m} (p_y \sigma_x - p_x \sigma_y)$$
Strength of Rashba term can be tuned by gate voltage!
On a straight wire of length **R**, $\mathcal{H}_{link} = -\frac{1}{2m^*} \nabla^2 - i \frac{\tilde{k}_{so}}{m^*} \hat{\mathbf{n}} \cdot [\boldsymbol{\sigma} \times \nabla]$

$$\mathbf{E} = E \hat{\mathbf{n}}$$

$$\mathcal{H}_{\text{link}} = \frac{\mathbf{k}^2}{2m^*} - \frac{\widetilde{k}_{\text{so}}}{m^*} [\hat{\mathbf{n}} \times \mathbf{k}] \cdot \boldsymbol{\sigma} = \frac{\mathbf{k}^2}{2m^*} - \mathbf{H}_{\text{so}} \cdot \boldsymbol{\sigma}$$

$$\mathcal{H}_{\text{link}} = \frac{\mathbf{k}^2}{2m^*} - \frac{\widetilde{k}_{\text{so}}}{m^*} [\boldsymbol{\sigma} \times \mathbf{n}] \cdot \hat{\mathbf{k}} = \frac{\left(\mathbf{k} - \widetilde{k}_{\text{so}} \boldsymbol{\sigma} \times \mathbf{n}\right)^2}{2m^*} + const.$$

Entin-Wohlman, Gefen, Meir, Oreg (1989, 1992): Spinor moving on wire "gains" a "phase factor" $e^{-ik_{so}[\mathbf{R} \times \hat{\mathbf{n}}] \cdot \boldsymbol{\sigma}}$



Spin field effect transistor

Electronic analog of the electro-optic modulator

Supriyo Datta and Biswajit Das School of Electrical Engineering, Purdue University, West Lafayette, Indiana 47907

Appl. Phys. Lett. 56 (7), 665

12 February 1990



$$\binom{1}{1}_{(45^{\circ} \text{ pot.})} = \binom{1}{0}_{(z \text{ pot.})} + \binom{0}{1}_{(z \text{ pot.})} \cdot P_0 \propto \left| (1 \ 1) \binom{e^{ik_z L}}{e^{ik_z L}} \right|^2 = 4 \cos^2 \frac{(k_1 - k_2)L}{2}$$



'Writing' on spinor (=Qubit): Spin filtering

Work with **mobile** electrons,

Generate **fully spin-polarized** current out of an unpolarized source



Can we imitate Datta and Das and use spin-orbit interactions to filter spins in mesoscopic wire networks?



Can spin polarization be generated in a **2-terminal** setup with **spin-orbit interaction** (SOI)?



Spin transmission between 2 terminals with time reversal symmetry?

$$|\psi^{L}\rangle = c^{in,L} |n\rangle + c^{out,L} |Tn\rangle \qquad |\psi^{R}\rangle = c^{in,R} |m\rangle + c^{out,R} |Tm\rangle$$

$$\begin{pmatrix} c^{out,L} \\ c^{out,R} \end{pmatrix} = S\begin{pmatrix} c^{in,L} \\ c^{in,R} \end{pmatrix} = \begin{pmatrix} r & t' \\ t & r' \end{pmatrix} \begin{pmatrix} c^{in,L} \\ c^{out,L} \end{pmatrix}$$

$$T |\psi^{L}\rangle = (c^{in,L})^{*} |Tn\rangle - (c^{out,L})^{*} |n\rangle \qquad T |\psi^{R}\rangle = (c^{in,R})^{*} |Tm\rangle - (c^{out,R})^{*} |m\rangle$$

$$\begin{pmatrix} (c^{in,L})^{*} \\ (c^{in,R})^{*} \end{pmatrix} = S\begin{pmatrix} -(c^{out,L})^{*} \\ -(c^{out,R})^{*} \end{pmatrix}$$

$$TT = -C^{out,R} = C^{in,R} = C^{in,R}$$

S unitary $S^{T}\begin{pmatrix} c^{in,L} \\ c^{in,R} \end{pmatrix} = -\begin{pmatrix} c^{out,L} \\ c^{out,R} \end{pmatrix}$ $S^{T} = -S$ $r^{T} = -r$ $r = \begin{pmatrix} 0 & \lambda \\ -\lambda & 0 \end{pmatrix}$

$$r^{\dagger}r = |\lambda|^2 I \qquad t^{\dagger}t = 1 - r^{\dagger}r$$

Same transmissions for both spin polarizations

Beenakker: scattering matrix is **self-dual**

J. H. Bardarson, J. Phys. A: Math. Theor. 41, 405203 (2008)



Can spin polarization be generated in

a 2-terminal setup with spin-orbit interaction?

NO, since SOI obeys time reversal symmetry

---- Kramer's degeneracy



MUST BREAK TIME REVERSAL SYMMETRY OR ADD TERMINALS

5 ways to obtain spin separation (filtering):

- 1) Break time reversal by Aharonov-Bohm flux
- 2) Break unitarity by leakage
- 3) Use 2 drain terminals
- 4) Use transient time-dependence
- 5) Add magnetization on leads

Filtering and analyzing mobile qubit information via Rashba–Dresselhaus– Aharonov–Bohm interferometers

Amnon Aharony,1,* Yasuhiro Tokura,2 Guy Z. Cohen,3,† Ora Entin-Wohlman,1,‡ and Shingo Katsumoto4

Spin transport through single or double diamond, With a magnetic flux – Aharonov-Bohm effect





AB interferometer with spin-orbit interactions



 $t|\chi_t\rangle = \mathcal{T}|\chi_{in}\rangle, \quad r|\chi_r\rangle = \mathcal{R}|\chi_{in}\rangle$

Single diamond



$$(\epsilon - \epsilon_u)\psi(u) = -\sum_v J_{uv}U_{uv}\psi(v)$$

$$\begin{split} U_{0b} &= \exp(i\alpha\sigma_1), \quad U_{b1} = \exp(-i\phi/2 - i\alpha\sigma_2), \\ U_{0c} &= \exp(-i\alpha\sigma_2), \quad U_{c1} = \exp(i\phi/2 + i\alpha\sigma_1), \end{split}$$

 $\mathbf{W} = \gamma_b U_b + \gamma_c U_c$

Tight-binding

$$\begin{aligned} (\epsilon - \epsilon_0) |\psi(0)\rangle &= -\left(\widetilde{U}_{0b} |\psi(b)\rangle + \widetilde{U}_{0c} |\psi(c)\rangle\right) - j |\psi(-1)\rangle \\ (\epsilon - \epsilon_1) |\psi(1)\rangle &= -\left(\widetilde{U}_{b1}^{\dagger} |\psi(b)\rangle + \widetilde{U}_{c1}^{\dagger} |\psi(c)\rangle\right) - j |\psi(2)\rangle, \\ (\epsilon - \epsilon_b) |\psi(b)\rangle &= -\left(\widetilde{U}_{0b}^{\dagger} |\psi(0)\rangle + \widetilde{U}_{b1} |\psi(1)\rangle\right), \\ (\epsilon - \epsilon_c) |\psi(c)\rangle &= -\left(\widetilde{U}_{0c}^{\dagger} |\psi(0)\rangle + \widetilde{U}_{c1} |\psi(1)\rangle\right). \end{aligned}$$

$$\begin{array}{c} \mbox{Eliminate}\\ \mbox{b and c} \end{array} & \longleftarrow \begin{array}{c} (\epsilon - \epsilon_0 - \gamma_b - \gamma_c) |\psi(0)\rangle = \mathbf{W} |\psi(1)\rangle - j |\psi(-1)\rangle,\\ (\epsilon - \epsilon_1 - \gamma_b - \gamma_c) |\psi(1)\rangle = \mathbf{W}^{\dagger} |\psi(0)\rangle - j |\psi(2)\rangle, \end{array}$$

 $\mathbf{W} \equiv \gamma_{0b1} U_{0b} U_{b1} + \gamma_{0c1} U_{0c} U_{c1}$

Non unitary: SO cage

Scattering theory

Electron from left:

$$\begin{split} |\psi(n)\rangle &= e^{ikna}|\chi_{in}\rangle + re^{-ikna}|\chi_r\rangle, \quad n \leq 0, \\ |\psi(n)\rangle &= te^{ik(n-1)a}|\chi_t\rangle, \quad n \geq 1, \end{split}$$

$$t|\chi_t\rangle = \mathcal{T}|\chi_{in}\rangle, \quad r|\chi_r\rangle = \mathcal{R}|\chi_{in}\rangle,$$

Transmission: $\mathcal{T} = 2ij \sin(ka) \mathbf{W}^{\dagger} (Y\mathbf{1} - \mathbf{W}\mathbf{W}^{\dagger})^{-1},$ Reflection: $\mathcal{R} = -\mathbf{1} - 2ij \sin(ka) X_1 (Y\mathbf{1} - \mathbf{W}\mathbf{W}^{\dagger})^{-1}.$

Basic idea: diagonalize transmission matrix

$$\mathcal{T} = t_{+} |\mathbf{n}'\rangle \langle \mathbf{n}| + t_{-}| - \mathbf{n}'\rangle \langle -\mathbf{n}|$$

Incoming spinor:
$$|\chi_{in}\rangle = c_+ |\hat{\mathbf{n}}\rangle + c_- |-\hat{\mathbf{n}}\rangle$$

Outgoing spinor:
$$t|\chi_t\rangle = c_+t_+|\hat{\mathbf{n}}'\rangle + c_-t_-|-\hat{\mathbf{n}}'\rangle$$

Full filtering if one eigenvalue vanishes!

$$t_{-} = 0 \qquad t|\chi_t\rangle = c_+ t_+ |\hat{\mathbf{n}}'\rangle \qquad T = T_+ |c_+|^2$$

Rashba spin orbit

Independent $\cos(\phi/2) = s^2 \sin(2\beta)$ of energy!







"Reading" spin information

Incoming electrons polarized, $|\chi_{in}\rangle\equiv|\hat{\mathbf{n}}_{0}
angle$

$$|c_{+}|^{2} = |\langle \hat{\mathbf{n}} | \hat{\mathbf{n}}_{0} \rangle|^{2} = \frac{1}{2} [1 + \hat{\mathbf{n}}_{0} \cdot \hat{\mathbf{n}}]$$

$$T = T_+ |c_+|^2$$

Can measure the projection of the incoming polarization on that of the filter





Can rotate the spins between the two diamonds



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New J. Phys. 15 125017 (2013)

Spin filtering in a Rashba–Dresselhaus–Aharonov–Bohm double-dot interferometer

> Shlomi Matityahu¹, Amnon Aharony^{1,2,3,6}, Ora Entin-Wohlman^{1,2,3} and Seigo Tarucha^{4,5}



Need to tune only 2 voltages!

Alternative: break unitarity by leakage

PHYSICAL REVIEW B 87, 205438 (2013)

Robustness of spin filtering against current leakage in a Rashba-Dresselhaus-Aharonov-Bohm interferometer

Shlomi Matityahu,¹ Amnon Aharony,^{1,2,3,*} Ora Entin-Wohlman,^{1,2,3} and Shingo Katsumoto⁴



Obtain same equations as before, but with shifted magnetic flux,

$$\widetilde{\phi} = \phi + \delta_c - \delta_b$$

 δ_h δ_c are phases of complex terms on each path

Phases behave like magnetic flux!

Leakage breaks time reversal symmetry!

No need for magnetic field

New alternative: use more than 2 terminals

PHYSICAL REVIEW B 95, 085411 (2017)

Spin filtering in all-electrical three-terminal interferometers

S. Matityahu,^{1,2,*} A. Aharony,^{1,3} O. Entin-Wohlman,^{1,3} and C. A. Balseiro^{4,5}



$$I_i^{(C)} = e \int \frac{dE}{2\pi\hbar} \sum_{j\neq i} [f_i(E) - f_j(E)] T_{i,j}^{(C)}(E),$$
$$I_i^{(S)} = \int \frac{dE}{4\pi\hbar} \sum_{j\neq i} [f_i(E) - f_j(E)] T_{i,j}^{(S)}(E),$$

Use same chemical potential on drains-Sufficient to solve 1 column in scattering matrix

$$\begin{aligned} \mathcal{R} &= -1 - 2iJ_0 Y \sin k_0 \left[Z - J_{0b} J_{0c} J \left(u + u^{\dagger} \right) \right]^{-1} \\ \mathcal{T}_1 &= 2iJ_0 \sin k_0 \left[Z - J_{0b} J_{0c} J \left(u + u^{\dagger} \right) \right]^{-1} U_{0b} \\ &\times \left(J J_{0c} u - X_c J_{0b} \right) , \end{aligned}$$

$$\begin{aligned} \mathcal{T}_2 &= 2iJ_0 \sin k_0 \left[Z - J_{0b} J_{0c} J \left(u + u^{\dagger} \right) \right]^{-1} U_{0c} \\ &\times \left(J J_{0b} u^{\dagger} - X_b J_{0c} \right) , \end{aligned}$$
(6)

$$r = -1 - \frac{2iJ_0 Y \sin k_0}{Z - 2J_{0b}J_{0c}J\cos\omega}$$

$$t_{\pm}^{(1)} |\chi_{t,\pm}^{(1)}\rangle = \frac{2iJ_0 \sin k_0 \left(JJ_{0c}e^{\pm i\omega} - X_c J_{0b}\right)}{Z - 2J_{0b}J_{0c}J\cos\omega} U_{0b} |\pm \hat{n}\rangle$$

$$t_{\pm}^{(2)} |\chi_{t,\pm}^{(2)}\rangle = \frac{2iJ_0 \sin k_0 \left(JJ_{0b}e^{\mp i\omega} - X_b J_{0c}\right)}{Z - 2J_{0b}J_{0c}J\cos\omega} U_{0c} |\pm \hat{n}\rangle$$

Obtain full polarization along both drains, provided that

$$\frac{JJ_{0c}}{J_{0b}} = |X_c| = \sqrt{y_c^2 + 2y_c J_0 \cos k_0 + J_0^2},$$
$$\tan \omega = \frac{J_0 \sin k_0}{y_c + J_0 \cos k_0}.$$

PHYSICAL REVIEW B 90, 165422 (2014)

Real-time dynamics of spin-dependent transport through a double-quantum-dot Aharonov-Bohm interferometer with spin-orbit interaction

Matisse Wei-Yuan Tu,¹ Amnon Aharony,^{2,3,*} Wei-Min Zhang,^{1,†} and Ora Entin-Wohlman^{2,3}





Yet another way: return to original Datta-Das SFET

THE JOURNAL OF PHYSICAL CHEMISTRY C Article

Effects of Different Lead Magnetizations on the Datta–Das Spin Field-Effect Transistor

Published as part of The Journal of Physical Chemistry virtual special issue "Abraham Nitzan Festschrift". A. Aharony,^{*,†,‡} O. Entin-Wohlman,[†] K. Sarkar,^{*,‡}[©] R. I. Shekhter,[§] and M. Jonson[§]



$$\dot{\boldsymbol{M}}^{L} = \dot{\boldsymbol{M}}_{z}^{L0} \hat{\boldsymbol{z}} + \dot{\boldsymbol{M}}_{\perp}^{L1} [(d_{z}\hat{\boldsymbol{d}} - \hat{\boldsymbol{z}}) - \sin(2\alpha)[\hat{\boldsymbol{d}} \times \hat{\boldsymbol{z}}] - \cos(2\alpha)(d_{z}\hat{\boldsymbol{d}} - \hat{\boldsymbol{z}})]$$

$$\dot{M}_{\perp}^{L1} = 2\pi J^2 [(\mathcal{N}_{R\uparrow} - \mathcal{N}_{R\downarrow})N_0^L - (\mathcal{N}_{L\uparrow} + \mathcal{N}_{L\downarrow})M_{0,z}^R]$$

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Related study: Chiral-induced spin selectivity (CISS)

Ron Naaman et al:

Chiral molecules can generate spin selectivity



PHYSICAL REVIEW B 93, 075407 (2016)

Spin-dependent transport through a chiral molecule in the presence of spin-orbit interaction and nonunitary effects

Shlomi Matityahu,^{1,*} Yasuhiro Utsumi,² Amnon Aharony,^{1,3,4} Ora Entin-Wohlman,^{1,3,4} and Carlos A. Balseiro^{5,6}

Our approach: scattering with helix between 2 leads

Tight binding hopping on helix

Interference: hopping between helix steps

Spin-orbit interaction





NO POLARIZATION WITH TIME REVERSAL SYMMETRY AND 2 TERMINALS! THEREFORE WE APPLY LEAKAGE. 39

Alternative: collect electrons at the end from the **2** last sites on the helix

AU AU Nickel

Spin filtering in all-electrical three-terminal interferometers

PHYSICAL REVIEW B 95, 085411 (2017)

S. Matityahu,^{1,2,*} A. Aharony,^{1,3} O. Entin-Wohlman,^{1,3} and C. A. Balseiro^{4,5}



Conclusions:

No spin splitting for 2 terminals plus spin orbit

Magnetic flux+Rashba \rightarrow spin filter, writing on mobile qubits

Double diamond = **Datta-Das** spin FET.

Can replace magnetic flux by **leakage.**

Can obtain spin splitting from transient currents.

Alternative: obtain full polarizations without magnetic field with 2 drains

Also: polarize one lead and get rotated spins in the other

